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A FINITE-CIRCUIT-ELEMENT CODE FOR MODELING THE DYNAMICS OF A GY--ETC(U)

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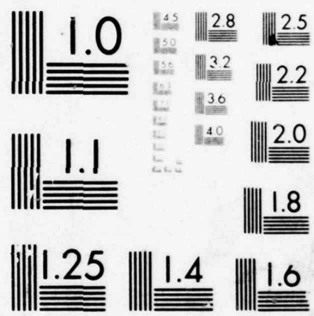

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A Finite-Circuit-Element Code for Modeling the Dynamics of a Gyrating Charged-Particle Beam

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A method is described for calculating the interaction between an imploding liner, a magnetically confined charged particle ring (Astron e-layer, ion ring) and a target plasma, based on the equations of the equivalent circuit. Expressing the electrodynamical behavior in terms of inductive coupling between circular current loops, so that changes in geometry and plasma parameters are described by changes in the induction coefficients, means that only ordinary differential equations arise, in contrast with fluid descriptions. Induced electron currents are conveniently included in the model. Application to a beam-target fusion system driven by the compression of an ion ring is described as a illustration of the utility of the technique.

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CONTENTS

A FINITE-CIRCUIT-ELEMENT CODE FOR MODELING THE COMPRESSION OF A GYRATING CHARGED-PARTICLE BEAM

I. INTRODUCTION

Over the last several years a great deal of interest has arisen in connection with the topic of gyrating intense ion beams.[1-3] A ring or cylindrical current layer is produced by the motion of the ions in the superposed background (quasiuniform) magnetic field and the poloidal self-field, with ring major radius R equal to the ion gyroradius. If the net current in such a configuration is strong enough, the direction of the field lines within the ring can be opposite that of the background field (Fig. 1). When the poloidal field on axis, $B_p = \mu_0 I / 2R$, exceeds the background field B_0, the field in the interior region is completely reversed.

Recently it has been proposed to increase the intensity of the neutral beams used to heat the plasma in 2xIIB and similar mirror devices in order to produce field reversal.[4] As pointed out by Baldwin and Rensink,[5] electric fields induced by the buildup of current tend to partially cancel the ion current. It is thus unclear that an initially unreversed configuration can become reversed, no matter how much ion current is added. Even if the configuration is compressed radially (by the action, e.g., of external coils, an imploding liner, or axial translation in a tank with converging metal walls), field reversal is problematic. The flux linking the ion ring tends to be conserved, and collisional diffusion only flattens the profiles.

Manuscript submitted July 19, 1978.

The present paper describes a code developed for treating the dynamics of a gyrating ion ring interacting with a background plasma and a (possibly imploding) metal liner. The code is called IPICAC (for Ion Beam-Plasma Interaction with Cylindrical Adiabatic Compression). It is two-dimensional (in r, z) and assumes axisymmetry, but does not employ finite-differences on a 2D grid to solve the dynamical problem. Instead, each portion of the system which carries current is regarded as part of a circular current loop. The beam is one such loop; the liner or wall may be approximated by several loops side by side. These current loops are coupled by their mutual inductances, and the dynamical behavior is determined through solution of the circuit equations. Thus the system is described by ordinary differential equations, rather then the partial differential equations of the usual magnetohydrodynamic treatment.

The principal difficulty in this approach lies in determining the inductances. These change as the geometry of the beam and liner changes, and have to be recalculated at every timestep. Unless some approximation is invoked to simplify them, no computational advantage results from the circuit theory technique. Fortunately, such an approximation is available in many charged-particle ring configurations of interest, namely that of large aspect ratio. That is, the major radius R_j of the jth current loop is taken to be large compared with its minor dimension and the separation in the $r-z$ plane between it and any other loop. It is not necessary but is often convenient to assume that resistance and current are distributed uniformly throughout the $r-z$ cross section of the loop. The latter may be of arbitrary shape, but is usually taken to be circular or rectangular.

In this conception, collisions between the ion beam and background plasma enter as a resistance (and possibly an Ohkawa current[6]). Plasma energy losses by radiation and convection also affect the beam dynamics through the inductances and the resistance. Consistent with this approach, the inertia of the various particle species is ignored (except in the centrifugal force), so that the beam and plasma remain in force balance with the wall currents.

The code described here was originally[1,7] developed for an ion-beam-plasma interaction problem related to but distinct from that of producing field reversal. We started with a ring of deuterium (D) ions assuming an already existing field-reversed geometry. The ring was compressed by implosion of the liner, and the thermonuclear energy production arising from collisions between the beam ions and T or He^3 target ions in the background plasma was studied. An attempt was made to balance the components of the system so that the collisional slowing-down of the beam ions just canceled their tendency to speed up because of angular momentum conservation. "Clamping" the beam in this way at the energy for which the beam-target reaction rate peaks ($\sim$150 keV for D-T reactions) maximizes Q, the ratio of the yield to the sum of liner and plasma energy. It was found however that even with optimized parameters, Q was limited to 10% or less. The reason was that the energy given up by beam ions in collisions, most of which went into electron heating, caused expansion of the toroidal beam-plasma system and reduced all of the number densities, and accordingly reduced the beam-target reaction rate. Presumably Q would increase if a method were found to cool the electrons and recycle their thermal energy.

Some results from this earlier work will be displayed for purposes of illustration, but the method is much more general in applicability. Instead of assuming a preexisting state of field reversal, one can employ the code to study its origin and development in time. This problem will not however, be addressed in the present paper, which is devoted to describing the code and some of the techniques employed in its implementation. The plan of the paper is as follows. In Section II we derive the equations of the circuit theory model of the beam-plasma-liner dynamical system. In Section III we discuss isentropic (lossless) compression of an ion ring and the role of the induced electron current in the resultant scaling. Collisions are described in Section IV. The implementation of conduction, particle transport processes and

other phenomena is discussed in Section V, and an example is described in Section VI. Our results are summarized in Section VII. A listing of the code is given in an Appendix.

II. LINER MOTION AND EQUIVALENT CIRCUIT EQUATIONS

It is natural to represent the ion beam (and the currents carried by the electrons and target ion) as a current loop. It is equally convenient, though perhaps less natural, to represent the axial current profile on the liner (and possibly on the driver coil) as a superposition of coaxial current loops. Each such loop constitutes an electrical circuit individually coupled to each of the others, and contains a self-inductance and a resistance (arising from charged-particle encounters in the case of the ring). The circuit elements vary in time as the geometry charges.

Thus it is possible to calculate the implosion dynamics to any desired degree of realism entirely by means of the equivalent circuit equations. This representation is, in fact, a type of "finite-element" simulation. The minimum number of such circuits required to describe electromagnetic implosions of the liner is one each for the driver, liner and ring. In this limit the equivalent circuit is that shown in Fig. 2.

The circuit equations take the form

$$\frac{d\Phi_j}{dt} = -\mathcal{R}_j I_j \tag{1}$$

where j runs over all current-carying loops in the system. For the circuit of Fig. 2, $j = d,l,r$ (signifying driver, liner, and ring, respectively). The flux threading the jth element is

$$\Phi_j = \sum_k \mathfrak{M}_{jk} I_k, \tag{2}$$

where $\mathfrak{M}_{jk}$ is the inductance coupling circuits j and k, and $\mathcal{R}_j$ is the resistance of the jth circuit. Equation (1) describes the evolution of Φ_j. Given a knowledge of the Φ_j and the induction coefficients $\mathfrak{M}_{jk}$, Eq. (2) then can be solved for the I_j by matrix inversion.

If the driver is static and energized only during the outermost portion of the cycle, we can make an additional simplification by restricting our attention to times when the liner and ring are far removed from the driver coil. Then j, k take on only the values l,r, and there are just two each of equations (1) and (2). The numerical results described and plotted below were obtained using this two-loop circuit. It should be clear, however, that most of the discussion which follows is independent of the number of loops employed. We have experienced no difficulty in implementing versions of the code where as many as ten loops are employed to simulate the current profile in the liner. It appears that it would be easy to generalize the method to multiple ion rings or single rings with multiple constituent current filaments.

The coefficients $\mathfrak{M}_{jk}$ are very easily calculated. Since the ring deforms freely, it tends to evolve so as to maximize its self-inductance, that is, toward a circular cross-section. Moreover, one wants to consider configurations where ring and liner are close together, to minimize the volume filled with magnetic energy. Thus all distances separating current-carrying filaments are small compared with the major radii R, R_l (Fig. 1). In this limit the self and mutual inductances can be calculated in the large-aspect-ratio approximation as

$$\mathfrak{L}, \mathfrak{M} \simeq \mu_o R\,[\ln(8R) - 2 - \ln\bar{D}], \tag{3}$$

where the average is over the current-carrying part of the cross section, and the minor diameter D satisfies $D << R$.

Using (3) we find that the self-inductance of the ring is given by

$$\mathfrak{L}_r \equiv \mathfrak{M}_{rr} = \mu_o R\,[\ln(8R/r) - 2 + \delta] \tag{4}$$

where r is the ring minor radius, and δ depends on the details of the assumed current profile. For example, if all the current is carried in a skin located at the minor radius, $\delta = 0$; if the current is uniformly distributed, $\delta = 0.25$; and if the ring looks like a Bennett pinch in cross-section, $\delta = 0.5$. Similarly, the self-inductance of a liner segment is approximately (assuming the current is carried on the inner surface)

$$\mathcal{L}_l \equiv \mathcal{M}_{ll} \simeq \mu_o R_l \left[\ln (\delta R/l) - 1/2\right] \tag{5}$$

where l is the length of the segment, assumed much larger then the thickness, and R_l is the inside radius; and

$$\mathcal{M}_{lr} = \mu_o (RR_l)^{1/2} \left\{ \ln \left\{ \frac{8(RR_l)^{1/2}}{\left[(R-R_l)^2 + (l/2)^2\right]^{1/2}} \right\} - 1 \right. \tag{6}$$

$$\left. - \left[(R_l - R)/(l/2)\right] \tan^{-1}\left[(l/2)/(R_l - R)\right] \right\}$$

More important than the exact forms of (5) and (6) (which depend on the cross sections assumed to describe the liner) is the fundamental geometrical requirement $M_{lr}^2 \leqslant L_r L_l$, with equality holding only if $R = R_l$. Since Eqs. (4-6) are approximate, this inequality must be enforced by means of an explicit interpolation; otherwise, the ring can pass right through the liner. The interpolation formula actually used is

$$\mathcal{M}'_{lr} = \mathcal{M}_{lr} + [(\mathcal{L}_r \mathcal{L}_l)^{1/2} - \mathcal{M}_{lr}] \left[1 + \left[(R_l - R)/r\right]^p\right], \tag{7}$$

where M' is the corrected value of the mutual inductance. The dynamical results are not very sensitive to the choice of p, which was taken to be 10 in the numerical calculation.

As is well known from electromagnetic theory, the force tending to change any coordinate θ on which an inductive coefficient $\mathcal{M}_{jk}$ depends is given by

$$F_{jk} = - I_j I_k \frac{\partial \mathcal{M}_{jk}}{\partial \theta}. \tag{8}$$

Employing (8) consistently with the definitions used for $\mathcal{M}_{jk}$ guarantees conservation of total energy, the magnetic portion of which is

$$W_M = \frac{1}{2} \sum_{j,k} \mathcal{M}_{jk} I_j I_k = \frac{1}{2} \sum_j I_j \Phi_j \tag{9}$$

Thus in carrying out numerical calculations, we determine the total force of the ring acting on the liner according to

$$F_l = - I_r \sum_j I_j \frac{\partial \mathfrak{M}_{rj}}{\partial R_l}, \tag{10}$$

where the summation runs over the ring and all segments of the liner; while the same expression with opposite sign yields the force with which the liner tends to hold the ring in place. The liner equation of motion is thus

$$M_l \ddot{R}_l = F_l \tag{11}$$

Similarly, the electromagnetic force acting to constrict the ring is given by Eq. (8) with $\theta = r$:

$$F_r = - I_r \sum_j I_j \frac{\partial \mathfrak{M}_{rj}}{\partial r}. \tag{12}$$

Most of the force F_r comes from the term containing $\mathfrak{M}_{rr} = \mathfrak{L}_r$. Because of the use of the interpolation formula, Eq. (7), however, there is a small contribution from the liner-ring mutual inductances.

III. ISENTROPIC COMPRESSION

It is possible to develop scaling laws in terms of which the liner motion and beam and plasma evolution are described by analytic expressions, provided we assume the absence of both fusion reactions and loss mechanisms. This model is not a useful starting point about which to perturb to describe a realistic reactor design, because the latter is quite sensitive to beam slowing and the heating resulting from production of charged fusion reaction products. It is, however, valuable in describing the dynamics in the absence of a target plasma, as well as guiding us in developing an intuition about the interdependence of various parts of the system.

If the liner is represented by J_l distinct current-carrying segments, there are $J_l + 1$ fluxes and $J_l + 11$ physical variables. In our numerical calculations we usually took $J_l = 1$. For this case the 12 physical quantities used to describe a dynamical state of the system are the fluxes Φ_l and Φ_r linking the liner and ring, respectively; R and R_l; the ring minor radius r, the total

numbers of beam and target ions, N_B and N_T, respectively; the beam, target and electron temperatures, T_B, T_T and T_e, respectively; and the mean azimuthal ion drift velocities v_B and v_T. To proceed, we write down all the conservation laws that are available. The conserved quantities are the magnetic flux threading the j th liner segment

$$\Phi_j = \sum_l \mathfrak{M}_{jl} I_l + \mathfrak{M}_{rj} = \Phi_j^o, \tag{13}$$

and that threading the ring,

$$\Phi_r = \mathfrak{L}_r I_r + \sum_l \mathfrak{M}_{rl} I_l = \Phi_r^o; \tag{14}$$

the specific angular momentum of beam ions,

$$R v_B = R^o v_B^o, \tag{15}$$

and of target ions,

$$R v_T = R^o v_T^o; \tag{16}$$

the total ion numbers for each species

$$N_B = N_B^o, \tag{17}$$

$$N_T = N_T^o; \tag{18}$$

and the beam, target and electron entropy functions:

$$T_B V^{\gamma-1} = T_B^o (V^o)^{\gamma-1}, \tag{19}$$

$$T_T V^{\gamma-1} = T_T^o (V^o)^{\gamma-1}, \tag{20}$$

$$T_e V^{\gamma-1} = T_e^o (V^o)^{\gamma-1}. \tag{21}$$

Here $V = 2\pi^2 R r^2$ is the volume of the beam/plasma ring. Superscripts (^{o}) indicate an initial or a reference state of the system (e.g., the state of maximum compression). To these equations must be added the condition of force balance on the ring in the direction of major and minor radius,

$$0 = I_r \sum_l I_l \frac{\partial \mathfrak{M}_{rl}}{\partial R} + \frac{1}{2} I_r^2 \frac{\partial \mathfrak{L}_r}{\partial R} + p \frac{\partial V}{\partial R} + \frac{N_B m_B v_B^2}{R} + \frac{N_T m_T v_T^2}{R} \tag{22}$$

and

$$0 = \frac{1}{2} I_r^2 \frac{\partial \mathcal{L}_r}{\partial r} + I_r \sum_l I_l \frac{\partial \mathfrak{M}_{rl}}{\partial r} + p \frac{\partial V}{\partial r}, \tag{23}$$

respectively. Here $p = k[N_B T_B + N_T T_T + N_e T_e] V^{-1}$ is the internal pressure in the ring (k is the Boltzmann constant), and the electron number is obtained from the condition of charge neutrality,

$$N_e = N_B Z_B + N_T Z_T, \tag{24}$$

where Z_α is the charge state of ion species α. The last two terms in eq. (22) are the centrifugal force terms derived from the circulation of the respective species; that corresponding to the target ions is usually negligible.

Equations (22) and (23) have been derived assuming that the ring inertia is negligible, i.e., that the ring repositions itself instantaneously in response to any change in the position of the liner. In addition, the electron mass has been set to zero systematically, as negligible in comparison with those of the ions. The ring current I_r satisfies

$$I_r = I_B + I_T + I_e, \tag{25}$$

where

$$I_B = \frac{N_B e Z_B v_B}{2\pi R}, \tag{26}$$

$$I_T = \frac{N_T e Z_T v_T}{2\pi R}, \tag{27}$$

and

$$I_e = -\frac{N_e e v_e}{2\pi R}. \tag{28}$$

Equations (1), (11) and (13-23) contribute a set of $12 + J_l$ fundamental algebraic equations in terms of the $12 + J_l$ physical quantities defining the state. [All the others are expressible in terms of these through Eqs. (2), (4-6), and (24-28).] Thus, specifying the state variables determines the evolution of the system completely. We rewrite the liner force equation as

$$\frac{d}{dt}(R_l \dot{R}_l) = \left\{ R_l^2 \dot{R}_l^2 (R_l^{-2} - R_l'^{-2}) + \sum_l \left[\frac{1}{2} I_l^2 \frac{\partial \mathcal{L}_l}{\partial R_l} + I_l I_r \frac{\partial \mathfrak{M}_{rl}}{\partial R_l} \right] / (2\rho L R_l) \right\} / \ln(R_l'^2 / R_l^2)$$

which parametrizes the dynamical history in terms of t. Equation (29) is derived by assuming conservation of the liner mass $M_l = 2\pi\rho L\,(R_l'^2 - R_l^2)$; ρ is the (uniform) liner density, L is the overall length, and R_l' is the outer liner radius.

Let us assume now that the electron current tending to neutralize I_B is zero. Then by conservation of angular momentum,

$$I_r = \frac{e}{2\pi R}\,(N_D \mathrm{v}_D + N_T \mathrm{v}_T) = \frac{e}{2\pi R^2}\,(N_D R \mathrm{v}_D + N_T R \mathrm{v}_T) \sim R^{-2}. \tag{30}$$

The minor radius force balance condition (23) reduces to

$$p = \frac{\mu_o}{4} \frac{R I_r^2}{V} \sim r^{-2} R^{-4} \tag{31}$$

Equations (19-21), weighted by the respective total numbers N_j, sum to the adiabatic law

$$p V^\gamma = \text{const.} \tag{32}$$

Taking $\gamma = 5/3$ and combining (31) and (32) yields

$$r \sim R^{7/4} \tag{33}$$

Hence the number densities for species $\alpha (\alpha = B, T, e)$, $n_\alpha = N_\alpha / V$, satisfy

$$n_\alpha \sim V^{-1} \sim R^{-9/2}, \tag{34}$$

and the poloidal field near the ring $B_p = \mu_o I_r / 2\pi r$ satisfies

$$B_p^2 \sim p \sim R^{-15/2} \tag{35}$$

We thus have a situation in which almost three-dimensional compression of the ring occurs as $R \simeq R_l$ is reduced. The poloidal field (35) rises almost as the inverse fourth power of R and the temperatures scale like $T \sim R^{-3}$.

At the other extreme, the motion of the liner may be such as to induce electron currents I_e totally neutralizing the change in ion current,

$$I_r \approx \text{const} \tag{36}$$

Going through the same steps as above, we find

$$r \sim R^{-5/4} \tag{37}$$

and hence

$$n_\alpha \sim V^{-1} \sim R^{3/2}, \tag{38}$$

and

$$B_p^2 \sim p \sim R^{5/2}. \tag{39}$$

In this limit the beam/plasma system *decompresses* during implosion, with n, p and B_p decreasing.

The actual result obtained by numerical solution of the equations naturally lies between these two extremes. The ring is always observed to compress, but at a rate slower then that given by Eqs. (34-35), and the scaling is not a power law in R. If $I_e = 0$ initially, the behavior tends to resemble the second model increasingly as turnaround is approached. The dependence of the degree of field reversal on the magnitude of the electron current induced during compression[5] explains why attempts to derive a scaling law for this parameter[3,8] do not appear to yield a simple result. There is, in fact no clear-cut way to predict the scaling without specifying the geometry of the compression.

IV. COLLISIONS

The electron thermal spread is assumed to be much larger than the thermal spread of either ion distribution or the relative drift between any two species. The average momentum transfer rate resulting from a collision between particles of species α and β is given by

$$m_\alpha \left(\frac{d\mathrm{v}_\alpha}{dt} \right)_\beta = - \nu_S^{\alpha/\beta} m_\alpha (\mathrm{v}_\alpha - \mathrm{v}_\beta) \tag{40}$$

where

$$\nu_s^{B/T} = \frac{4\pi Z_B^2 Z_T^2 e^4 \,(1 + m_B/m_T) \ln \Lambda \; n_T}{m_B^2 \, V_{BT}^3}; \tag{41}$$

$$\nu_s^{\alpha/e} = \frac{4\sqrt{2\pi}}{3} \frac{Z_\alpha^2 e^4 (1+m_\alpha/m_e) m_e^{3/2} n_e \ln \Lambda}{m_\alpha^2 (kT_e)^{3/2}}, \tag{42}$$

$\alpha = B, T$, and, from conservation of momentum,

$$n_\alpha \, m_\alpha \, \nu_s^{\alpha/\beta} = n_\beta \, m_\beta \, \nu_s^{\beta/\alpha} \tag{43}$$

Here $\ln \Lambda$ is the form of the usual Coulomb logarithm appropriate to the species pair α, β, and $\mathrm{v}_{\alpha\beta} = |\mathrm{v}_\alpha - \mathrm{v}_\beta|$. Correspondingly, the average temperature rate of change resulting from a collision is.

$$k\left(\frac{dT_B}{dt}\right)_T = \frac{8\pi}{3} \frac{Z_B^2 Z_T^2 e^4}{m_B} \frac{n_T \ln\Lambda}{V_{BT}}; \tag{44}$$

$$k\left(\frac{dT_T}{dt}\right)_B = \frac{8\pi}{3} \frac{Z_B^2 Z_T^2 e^4}{m_T} \frac{n_\beta \ln\Lambda}{V_{BT}}, \tag{45}$$

for ion-ion encounters, and

$$k\left(\frac{dT_\alpha}{dt}\right)_e = \frac{8\sqrt{2\pi}}{3} \frac{Z_\alpha^2 e^4 \sqrt{m_e}\, n_e \ln\Lambda}{m_\alpha (kT_e)^{3/2}} k(T_e - T_\alpha), \tag{46}$$

$\alpha = B, T$, for ion-electron encounters, with the remaining rates $(dT_e/dt)_\alpha$ defined so as to satisfy conservation of energy.

Consideration of the magnitudes of these rate formulas reveals the following general features: (*i*) both electron and target ions contribute significantly to the rate at which beam ions slow down; (*ii*) the *relative* velocity with which beam ions move with respect to the target ions is chiefly affected by $B-T$ collisions, because electron collisions act in the same sense (as a drag) on both ion species; (*iii*) thermalization of the beam also results principally from collisions with target ions.

On the basis of these generalizations, we can estimate the relative slowing down of beam and target ions through collisions as

$$\begin{aligned} \frac{d}{dt}(\mathrm{v}_B - \mathrm{v}_T)_{\text{coll}} &\simeq -(\nu_s^{B/T} - \nu_s^{T/B})(\mathrm{v}_B - \mathrm{v}_T) \\ &\equiv -\nu_s(\mathrm{v}_B - \mathrm{v}_T). \end{aligned} \tag{47}$$

For the usual case where the target ion mass density substantially exceeds that of the beam, $n_T m_T >> n_B m_B$, Eq. (47) implies

$$\nu_s \simeq \nu_s^{B/T}. \tag{48}$$

at the same time, the adiabatic compression produced by the imploding liner tends to cause both ion species to accelerate in the azimuthal direction according to

$$\left(\frac{d\mathrm{v}_\alpha}{dt}\right)_{\mathrm{adiab}} = -\mathrm{v}_\alpha \frac{\dot{R}}{R} \simeq -\mathrm{v}_\alpha \frac{\dot{R}_l}{R_l} \tag{49}$$

Taking the difference between the beam and target equation (49) yields

$$\frac{d}{dt}(\mathrm{v}_B - \mathrm{v}_T)_{\mathrm{Adiab}} \simeq -\frac{\dot{R}_l}{R_l}(\mathrm{v}_B - \mathrm{v}_T). \tag{50}$$

Eqs. (47 and (50) give for the net time rate of change of the relative velocity

$$\frac{d}{dt}(\mathrm{v}_B - \mathrm{v}_T) \simeq -(\dot{R}_l/R_l + \nu_s)(\mathrm{v}_B - \mathrm{v}_T) \tag{51}$$

The condition that this relative velocity be a constant is thus

$$\dot{R}_l/R_l = -\nu_s \tag{52}$$

When Eq. (57) is satisfied, the beam is said to be *clamped*[10]. With a tritium target there is an advantage in clamping the beam at a relative energy $\epsilon = \frac{1}{2} m_B \mathrm{v}_{BT}^2 \sim 150$ keV which maximizes the reaction rate for D-T fusion.

Clamping is of course accompanied by a monotonic increase in thermal energy according to Eqs. (44-45). The ion thermal energy density $w_{th}^i = \frac{3}{2} k(n_B T_B + n_T T_T)$ increases as a result of ion-ion collisions at a rate

$$\frac{dw_{th}^i}{dt} = \frac{4\pi Z_B^2 Z_T^2 e^4 \, n_B n_T \ln\Lambda}{\mathrm{v}_{BT}} \left(\frac{1}{m_B} + \frac{1}{m_T}\right) \tag{53}$$
$$= \nu_s^{B/T} n_B m_B \mathrm{v}_{BT}^2$$

Using (48), we see by comparison of (52) and (53) that the time scale for implosions of the liner is comparable to that for heating up the ion beams. The electron heating rate can be even faster.

Note that if ν_s were approximately constant, the clamping condition (52) would imply an exponential decrease in R_l with time. As this is not realizable, clamping evidently cannot be maintained close to turnaround.

In differencing the equations in the code, we found it convenient to use as dependent variables quantities that are approximately conserved. Thus instead of T_α we used the entropy functions [Eqs. (19-21)], which now satisfy equations of the form

$$\frac{d}{dt}\,(T_\alpha V^{\gamma-1}) = V^{\gamma-1} \sum_\beta \nu_T^{\alpha/\beta}\,(T_\beta - T_\alpha), \tag{54}$$

where the $\nu_T^{\alpha/\beta}$ are defined as the rates in Eqs. (44-46). Similarly, the slowing-down rates enter as

$$\frac{d}{dt}\,(R\,\mathrm{v}_\alpha) = R \sum_\beta \nu_s^{\alpha/\beta}(\mathrm{v}_\beta - \mathrm{v}_\alpha). \tag{55}$$

V. OTHER DISSIPATIVE PROCESSES

Collisions, discussed in section IV, can transform directed energy into thermal energy. Although essential for clamping, they may be deleterious if they (i) increase the ratio of beam ion gyroradius to ring thickness excessively; (ii) cause too much of the liner energy to go into pumping up the target plasma; or (iii) lead to premature loss of confinement as a result of decrease of beam current below that needed for field reversal. In addition, the following loss processes can remove energy from the system entirely: radiation, heat conduction along field lines, particle diffusion across lines, charge exchange with impurities, and ohmic heating within the liner. The last of these can have a second, more serious consequence: finite resistivity gives rise to diffusion of field lines through the liner, untrapping the magnetic flux which holds the ring at a safe distance from the liner.

Radiation processes are modeled by adding loss terms to the expression (55) for the time rate of change of the electron entropy function. For bremsstrahlung and synchrotron (cyclotron) radiation we have the terms

$$\frac{d}{dr}\left(V^{\gamma-1}\, T_e\right)_{br} = -\, V^{\gamma-1} \times\ 5.35 \times 10^{-24}\,(N_D + N_T Z_T^2)\ T_e^{1/2} \tag{56}$$

and

$$\frac{d}{dt}\,(V^{\gamma-1} T_e)_{cyc} = -\, V^{\gamma-1} \times\ 3.98 \times 10^{-16}\,\frac{\beta^2 \bar{B}_p^2}{1-\beta^2}, \tag{57}$$

where T is given in eV, $\beta^2 = \frac{3}{2}\,kT_e/m_e c^2$ and $B_p = \mu_o I_r/2\pi r$. In the spirit of the circuit-theoretical approach (wherein the ring is a macroscopic circuit element with certain lumped parameters derived from microscopic processes), the radiation rates are calculated by averaging the field strength over the ring cross-section.

In the same fashion, thermal conduction losses can be treated by writing

$$\frac{d}{dt}\left(V^{\gamma-1}\, T_\alpha\right)_{cond} = -\, V^{\gamma-1}\, 4\pi^2 Rr\, \kappa_\alpha\, (T_\alpha/r) = -\, V^{\gamma-1}\, 4\pi R\, \kappa_\alpha T_\alpha, \tag{58}$$

where κ_α is the average cross field thermal conduction of species α. The fastest thermal loss process is that associated with the target ions, $\alpha = T$. Furthermore, thermal equilibration, alpha-particle heating, etc., can be included in an average sense in the same form.

Finally, particle losses can be estimated simply by assuming smeared-out density profiles according to some law like the Bennett pinch. If a given profile extends past the position of the separatrix, located at average minor radius $r = r_s$, that portion of the particles located at $r > r_s$ is lost. A simple calculation then gives the loss rate as the rate at which particles "fall over the edge." Thus we find

$$\left(\frac{dN_\alpha}{dt}\right)_{diff} = -\,\frac{Gr^2}{r_S^2}\,\nu_s^\alpha N_\alpha, \tag{59}$$

where G is a geometrical factor (equal to 12 for a Bennett profile) which decreases as the assumed profile becomes more localized, and ν_s^α is the total scattering rate for species α.

VI. A NUMERICAL EXAMPLE

Using the equations and techniques described in Section II-V, we consider the following situation. A liquid lithium liner (density $\rho = 0.54$ g/cm^3) of length $L = 13.5$ cm and inner and

outer radii 31.59 cm and 48.43 cm implodes with velocity 3×10^4 cm/s on a fully ionized D-He^3 ring with major and minor radii of 30 cm and 0.758 cm, respectively. The initial target ion number densities are $n_{He^3} = 2n_D = 3.59 \times 10^{16}$ cm^{-3}. The temperatures are $T_D = 23.7$ keV, $T_{He^3} = 1$ keV and $T_e = 10$ keV. The deuterium current is 1.52 MA, twice the electron back current. These numbers are chosen to give a beam ion streaming energy of 550 keV and a poloidal field of 200 kG, with beam clamping. Since the emphasis was on determining Q, only the part of the evolution in the vicinity of liner turnaround was considered, and the early-time conditions giving rise to these parameters were not investigated.

Figure 3 shows how the beam and liner radii change in time. Note that the separation increases, a reflection of the increase in beam minor radius (Fig. 4). Correspondingly the number densities (Fig. 5) drop, level off as collisional heating and compression come into balance, then drop again in the expansion (decompression) stage, and the poloidal field (Fig. 6) decreases, increases, then decreases monotonically after turnaround. The various forms of energy (magnetic, liner kinetic, ion directed, and thermal) are plotted in Fig. 7, along with the fusion yield. Figure 8 shows how the component temperatures increase near turnaround, the evident irreversibility being a consequence of collisions.

Running time on the calculation using an IBM 360/168 was 91 seconds, of which about a quarter was required for diagnostics. Using ten current loops to represent the liner current profile approximately doubles the running time, since roughly twice as many differential equations have to be solved [matrix inversion of Eq. (2) does not add any substantial amount to the total]. It turns out to be convenient in writing the code to make extensive use of nested sequences of statement functions in redetermining force balance on each time step, and most of the running time is expended in this task.

A variety of prescriptions are possible for defining the initial conditions. The main thing is to insure that they be neither over- nor underdetermined. When working with multiple liner

current loops, we arbitrarily imposed the condition that the flux threading all the loops be the same. Though straightforward, this is unlikely to be a good approximation in the late stages of the implosion if finite liner resistivity is modeled.

VII. CONCLUSIONS

We have presented a new numerical technique for solving problems involving the dynamics of charged particle rings. Its principle advantage is that it is couched in circuit-theoretical terms, obviating the need for solution of partial differential equations. Because of its adaptation to the physics and geometry of such problems, the method can be implemented with only a small number ($\sim$ 10) of current carrying elements. In effect, it replaces the uniform or quasi-uniform mesh of the standard 2D finite-difference technique with a highly nonuniform "mesh" of circuit elements, located optimally to reflect the relevant physics.

The code has been applied to calculations of the thermonuclear yield and other characteristics of a beam-target fusion device. The particular concept for which the code was originally developed turns out to be disappointing in terms of its efficiency as a reactor (the examples of Section VI yielded $Q \approx 3.2\%$), and also appears to be unstable to kink modes[11]; however it may have non-fusion applications. It is clear that the code can be applied to a variety of axisymmetric situations involving field reversal and changes of system geometry, and therefore is potentially of wider utility.

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```
C
C      "ION BEAM-PLASMA INTERACTION CLAMPED THROUGH AXIAL COMPRESSION"
C
C      THIS PROGRAM CALCULATES COMPRESSION, HEATING AND BURN IN AN AXIALLY
C      COMPRESSED BEAM-TARGET PLASMA.  THE BEAM IS A RING OF D IONS, THE
C      TARGET A MIXTURE OF COLD TRITIUM AND HOT ELECTRONS.  THE CONFINING
C      MAGNETIC FIELD BUILDS UP OWING TO COMPRESSION PRODUCED BY AN
C      IMPLODING LITHIUM LINER (LINUS), DRIVEN ELCTROMAGNETICALLY BY
C      EXTERNAL THETA-PINCH WINDINGS.  PARAMETERS ARE CHOSEN SO THAT
C      COMPRESSION, COLLISIONAL SLOWING OF THE D BEAM, AND LOSSES BALANCE,
C      PRODUCING A SITUATION IN WHICH THE BEAM IS CLAMPED FOR THE ENTIRE
C      DURATION OF THE COMPRESSION.
C      D. L. BOOK, P. J. TURCHI & D. L. STEIN
C
          IMPLICIT REAL*8 (A-I, K-Z), INTEGER*4 (J), REAL*4 (S)
          LOGICAL*4 COLL, LOSS, BURN, FORWRD
          COMMON/ NEWTON/ DT0,JITER
          COMMON /QBLOCK/ Q(25)
          COMMON/ ARRAY/ Y(25), DY(25), Y0(25)
          EXTERNAL DERIV
          NAMELIST /LINER/ R10, R20, RW, B20, RHO, OMEGA, LNGTHO, ETA,
     1        SPHT, TL, PCA
          DATA R10, R20, RW, B20, RHO, OMEGA, LNGTHO, ETA, SPHT, TL, PCA
     1        / 2.06D2, ?.126D2, 3.D2, 0.D0, .54D0, 0.D0, 13.5D0,
     2        45.2D-7, 6.D7, 5.D2, .1D0 /
          NAMELIST /PLASMA/ RMAJOR, WD, TD, TE, TT, BP, VRATIO, NRATIO,
     1        IRATIO
          DATA RMAJOR, WD, TD, TE, TT, BP, VRATIO, NRATIO, IRATIO /
     1        2.D2, 7.D2, 1.D2, 1.D2, 1.D2, 5.D5, .5D0, 1.D0, .5D0 /
          NAMELIST /LOGICL/ COLL, LOSS, BURN, FORWRD
          DATA COLL, LOSS, BURN, FORWRD / .TRUE., .TRUE., .TRUE.,
     1        .FALSE. /
          NAMELIST /CNTROL/ DT, DTPLOT, DTFILM, DTDUMP, TLAST, T, TPLOT,
     1        TFILM, TDUMP
          DATA DT, DTPLOT, DTFILM, DTDUMP, TLAST, T, TPLOT, TFILM, TDUMP
     1        /1.D-5, 1.D-4, 1.D-3, 1.D-2, .1D0, 0.D0, -1.D-12, -1.D-12,
     1        -1.D-12 /
          CALL INDUMP
          CALL CRDUMP(40,10)
C
C      INITIALIZE.
C
   10     READ (5,LINER,END=100)
          READ (5,PLASMA,END=100)
          READ (5,LOGICL,END=100)
          READ (5,CNTROL,END=100)
          PRINT 98
          DO 11  J = 1, 5
   11     PRINT 99
          WRITE (6,LINER)
          WRITE (6,PLASMA)
          WRITE (6,LOGICL)
          WRITE (6,CNTROL)
          DO 15  J = 1, 25
   15     Q(J) = 0.D0
          JITER=0
          DTMAX = DT
          DT0=DT
          CALL INPUT(T,
     1        R10, R20, RW, B20, RHO, OMEGA, LNGTHO, ETA, SPHT, TL, PCA,
     2        RMAJOR, WD, TD, TE, TT, BP, VRATIO, NRATIO, IRATIO,
     3        COLL, LOSS, BURN, FORWRD,
     4        DTPLOT, DTFILM, DTDUMP, TLAST, TPLOT, TFILM, TDUMP)
```

```
C
C     PERFORM DUMPS, DIAGNOSTICS AT APPROPRIATE INTERVALS.
C
   20       IF (T .LT. TDUMP)  GO TO 30
            TDUMP = TDUMP + DTDUMP
   30       IF (T .LT. TFILM)  GO TO 40
            TFILM = TFILM + DTFILM
   40       IF (T .LT. TPLOT)  GO TO 50
            TPLOT = TPLOT + DTPLOT
            CALL DERIV(T,JTOTAL)
            IF (T .EQ. 0.D0)  PRINT 90
            IF (T .NE. 0.D0)  PRINT 91, T
            DATA SY, SYO, SDY, SLI, SRING, SPLAS, SNU, SCNU, SDATA /
     1          'Y   ', 'YO  ', 'DY  ', 'LI  ', 'RING', 'PLAS', 'NU  ',
     2          'CNU ', 'DATA' /
            CALL DSPLAY(SY)
            CALL DSPLAY(SYO)
            CALL DSPLAY(SDY)
            CALL DSPLAY(SLI)
            CALL DSPLAY(SRING)
            CALL DSPLAY(SPLAS)
            CALL DSPLAY(SNU)
            IF ( T .GT. 0.D0)  GO TO 50
            CALL DSPLAY(SCNU)
            CALL DSPLAY(SDATA)
   50       CONTINUE
C
C     ADVANCE VARIABLES ONE TIMESTEP.
C
            JITER=0
            IF (Y(3) .NE. 0.D0)  DT = DMIN1(DTMAX, .05D0*DABS(Y(2)/Y(3)))
            DT0=DT
            CALL INT(JTOTAL,T,DERIV,DT)
            IF (T .LT. TLAST)  GO TO 20
            GO TO 10
   90       FORMAT('0', 'INITIAL DISPLAY, INCLUDING TABULATED CONSTANTS:')
   91       FORMAT('1', 'DISPLAY AT TIME T = ', 1PD10.4, ' SEC:')
   98       FORMAT('1', 60X, 'I P I C A C' // ' ', 35X, 'ION BEAM-',
     1          'PLASMA INTERACTION CLAMPED THROUGH AXIAL COMPRESSION')
   99       FORMAT('+', 35X,                       'I            ',
     1          'P        I           C              A      C          ')
  100       RETURN
            E N D
C
      SUBROUTINE DERIV(TIME,JTOTAL)
            IMPLICIT REAL*8 (A-I, K-Z), INTEGER*4 (J), REAL*4 (S)
            INTEGER*4 MOD
            DIMENSION RLINER(5), EMAG1(5)
            DIMENSION ZA(3,3), ZR(3)
            DIMENSION RMAJ1(4),RMAJ2(4),RMIN1(4),RMIN2(4)
            COMMON / NEWTON /DT, JITER
            COMMON / INCO / MU, KMU
            COMMON/ RADII/ RMINOR,RMAJOR,RJ
            COMMON / PBLOCK / P
            COMMON/ COEFF/ KCF,NDTOT, RVD
            COMMON / CURREN / ILINER(3)
            COMMON/ LENGTH/  LNGTH(5), LHLSQ(5)
            COMMON / INDEX / JMAX, JMAX1
            COMMON/ FLUX/ PHI,PSI1(5)
            COMMON/ ARRAY/ Y(25), DY(25), Y0(25)
            LOGICAL*4 NOCOLL, NOLOSS, NOBURN,FORWRD
            DIMENSION SLINE(33)
C
C     THE FOLLOWING STATEMENT FUNCTIONS ARE USED BELOW IN SOLVING FOR
C     RMAJOR AND RMINOR BY REQUIRING THAT THE FORCES IN THE CORRESPONDING
C     DIRECTIONS VANISH.  HERE A, B, C ARE RESPECTIVELY THE MINOR AND
C     MAJOR RADII OF THE RING AND THE LINER INSIDE RADIUS, ALL IN CM.
```

```
C
      LR(A,B) = KMU*B*(DLOG(8.D0*B/A) - 2.D0 + KD)
      DALR(A,B) = -KMU*B/A
      DBLR(A,B) = KMU*(DLOG(8.D0*B/A) - 1.D0 + KD)
      LL(C,D)=MU*C*(DLOG(16.D0*C/D) - 0.5D0)
      DCLL(C,D)=MU*(DLOG(16.D0*C/D) + 0.5D0)
      LN(D,F)=0.5D0*(D/F)*DLOG(D)
      FAC(D,F)=(((F-D)**2)/(2.D0*D*F))*DLOG(DABS(F-D))
      MLL(C,D,F)=MU*C*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)-0.5D0)
      DCMLL(C,D,F)=MU*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)+0.5D0)
      MLR(B,C,F)=MU*DSQRT(B*C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2 +
     1     F)) - 1.D0 - ((B-C)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      DBMLR(B,C,F)=.5D0*MU*DSQRT(C/B)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1     +F))+((C-3.D0*B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      DCMLR(B,C,F)=.5D0*MU*DSQRT(B/C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1     +F))+((3.D0*C-B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      MLRS(A,B,C,D,F)=MLR(B,C,F)+(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))
     1     /(1.D0+((C-B)/A)**P)
      DAMLRS(A,B,C,D,F)=(.5D0*DALR(A,B)*DSQRT(LL(C,D)/LR(A,B))+(P/A)
     1    *((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))/(1.D0+
     2     ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
      DBMLRS(A,B,C,D,F)=DBMLR(B,C,F)+(.5D0*DBLR(A,B)*DSQRT(LL(C,D)/
     1     LR(A,B)) + (P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-
     2    MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
      DCMLRS(A,B,C,D,F)=DCMLR(B,C,F)+(.5D0*DCLL(C,D)*DSQRT(LR(A,B)/
     1     LL(C,D))-(P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B  )*LL(C,D))-
     2    MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
      NKT(A,B) = PVG/(KVOL*A*A*B)**GM1
      SIGMA(E) = 1.D-24*(A5 + A2/(1.D0 + (A3*E - A4)**2))/(E*
     1     (DEXP(A1/DSQRT(E)) - 1.D0))
      DATA ETOTO / 0.D0 /
C
C     TIME (INDEPENDENT VARIABLE) HAS UNIT DERIVATIVE.
C
      DY(1) = 1.D0
C
C     REPLACE SUBSCRIPTED QUANTITIES WITH MORE FAMILIAR NOTATION.
C
      R1SQ = Y(2)
      RU = Y(3)
      PSI2 = Y(5)
      RVD = Y(7)
      RVT = Y(8)
      TDVGM1 = Y(9)
      TEVGM1 = Y(10)
      TTVGM1 = Y(11)
      NDTOT = Y(12)
      NTTOT = Y(13)
      EOHMIC = Y(14)
      ERAD = Y(15)
      NCOUNT = Y(16)
      PVGPRD = Y(17)
      BURNUP = Y(18)
      DO  5 J=1,JMAX1
    5 PSI1(J)=Y(19+J)
      PHI=Y(19+JMAX)
C
C     FIND SOME OF THE QUANTITIES NEEDED TO DESCRIBE THE LINER DYNAMICS.
C
      RSQUSQ = RU*RU
      R2SQ = R1SQ + R0SQ
      R1 = DSQRT(R1SQ)
      R2 = DSQRT(R2SQ)
      U1 = RU/R1
      U2 = RU/R2
```

```
      B1 = KFLUX*PSI1(1)/R1SQ
      B2 = KFLUX*PSI2/(RWSQ - R2SQ)
      RLOG = DLOG(R2SQ/R1SQ)
      DRSQ = R10SQ - R1SQ
      R12SQ = 1.D0/R1SQ - 1.D0/R2SQ
      PMAG1 = KP*B1*B1
      PMAG2 = KP*B2*B2
      U1BYR1 = U1/R1
C
C     NOW DO ION RING DYNAMICS AND PLASMA PROCESSES.  USE ALL THE
C     AVAILABLE ALGEBRAIC RELATIONS BEFORE COMPUTING ANY DERIVATIVES.
C     THE RING MAJOR AND MINOR RADII ARE FOUND USING NEWTON'S METHOD TO
C     SOLVE THE EQUATIONS FOR FORCE BALANCE IN THE RING, GIVEN THE FLUXES
C     WHICH ARE ENCLOSED BY THE RING AND LINER (PHI AND PSI1,
C     RESPECTIVELY), AND USING HANDBOOK FORMULAS FOR SELF- AND MUTUAL
C     INDUCTANCES.  MKS UNITS ARE USED IN THIS PORTION OF THE CODE.
C
C     EXTRAPOLATE FROM LAST TWO TIME STEPS TO GET GOOD INTIIAL GUESSES
C     FOR RMAJOR, RMINOR (USED ONLY ON EVEN STEPS OF R-K-G).
C
      IF (JITER .NE. 0)  GO TO 1
      JSTEP = 0
      DTOLD = DTNEW
      DTNEW = DT
      WT1 = -DTNEW/DTOLD
      WT2 = 1.D0 - WT1
    1 CONTINUE
C
      JSTEP = JSTEP + 1
      IF (MOD(JSTEP,2) .NE. 0)  GO TO 2
      RMAJOR = WT1*RMAJ1(JSTEP) + WT2*RMAJ2(JSTEP)
      RMINOR = WT1*RMIN1(JSTEP) + WT2*RMIN2(JSTEP)
    2 CONTINUE
C
      JIT = 0
      A = RMINOR
      B = RMAJOR
      RJ = R1
      PVG = KNKT*(NDTOT*(TDVGM1 + ZD*TEVGM1) + NTTOT*(TTVGM1 +
     1      ZT*TEVGM1)) + 1.D-7*PVGPRD
   10 CONTINUE
      DAFA=(FA(A+D/2.D0,B,RJ,NKT(A+D/2.D0,B))-FA (A-D/2.D0,B,RJ,
     1      NKT(A-D/2.D0,B)))/D
      DAFB=(FB(A+D/2.D0,B,RJ,NKT(A+D/2.D0,B))-FB (A-D/2.D0,B,RJ,
     1      NKT(A-D/2.D0,B)))/D
      DBFA=(FA(A,B+D/2.D0,RJ,NKT(A,B+D/2.D0))-FA(A,B-D/2.D0,RJ,
     1      NKT(A,B-D/2.D0)))/D
      DBFB=(FB(A,B+D/2.D0,RJ,NKT(A,B+D/2.D0))-FB(A,B-D/2.D0,RJ,
     1      NKT(A,B-D/2.D0)))/D
      DET = DAFA*DBFB - DAFB*DBFA
      FMINOR = FA(A,B,RJ,NKT(A,B))
      FMAJOR = FB(A,B,RJ,NKT(A,B))
      DA = (DBFB*FMINOR - DBFA*FMAJOR)/DET
      DB = (DAFA*FMAJOR - DAFB*FMINOR)/DET
      A = A - DA
      B = B - DB
      JIT=JIT+1
      IF (DABS(DB) .GT. RTEST*B .OR. DABS(DA) .GT. RTEST*A) GO TO 10
      RMINOR = A
      RMAJOR = B
      JITER=JITER+JIT
      ITER = DFLOAT(JITER)
      IF (MOD(JSTEP,2) .NE. 0)  GO TO 12
      RMAJ1(JSTEP) = RMAJ2(JSTEP)
      RMAJ2(JSTEP) = RMAJOR
      RMIN1(JSTEP) = RMIN2(JSTEP)
      RMIN2(JSTEP) = RMINOR
   12 CONTINUE
```

```
C
C     DEFINE THE FOLLOWING FOR DIAGNOSTIC PURPOSES.
C
           LL1=LL(RJ,LNGTH(1))
           LL2=LL(RJ,LNGTH(2))
           ML12=MLL(RJ,LNGTH(2),LNGTH(1))
           MLRS1=MLRS(RMINOR,RMAJOR,RJ,LNGTH(1),LHLSQ(1))
C
C     EVERYTHING ELSE CAN NOW BE CALCULATED.
C.
           LRING = LR(RMINOR, RMAJOR)
           LLINER=LL(RJ,LNGTH(JMAX1))
           IRING=ILINER(JMAX)
           VD = RVD/RMAJOR
           VT = RVT/RMAJOR
           VOLUME = KVOL*RMAJOR*RMINOR**2
           ND = NDTOT/VOLUME
           NT = NTTOT/VOLUME
           NETOT = NDTOT*ZD + NTTOT*ZT
           NE = NETOT/VOLUME
           ID = KID*NDTOT*VD/RMAJOR
           IT = KIT*NTTOT*VT/RMAJOR
           IE = IRING - ID - IT
           VE = RMAJOR*IE/(KIE*NETOT)
C
C     HAVING COMPUTED THE MAGNETODYNAMIC PARAMETERS, WE CAN WRITE DOWN
C     THE EQUATIONS OF MOTION OF THE LINER.
C
           DY(2) = 2.D0*RU
           JMAX2=JMAX1-1
           FORCE=0.D0
           DO 14 J=1,JMAX1
           FORCE=FORCE+ILINER(J)*(0.5D0*ILINER(J)*DCLL(RJ,LNGTH(J))
     1         +IRING*DCMLRS(RMINOR,RMAJOR,RJ,LNGTH(J),LHLSQ(J)))
           IF(J.EQ.JMAX1)  GO TO 14
           JP1=J+1
           DO 13 JJ=JP1,JMAX1
13         FORCE=FORCE+ILINER(J)*ILINER(JJ)*DCMLL(RJ,LHLSQ(JJ),LHLSQ(J))
14         CONTINUE
           P1 = KF*FORCE/RJ
           P2 = PMAG2
           DY(3) = (RSQUSQ*R12SQ + OMEGA**2*(R0SQ + DRSQ*(2.D0*RLOG +
     1         DRSQ*R12SQ)) + 2.D0*(P1 - P2)/RHO)/RLOG
C
C     FIND TEMPERATURES FROM PRODUCT OF V TO POWER GAMMA - 1 AND T.
C
           VGM1 = VOLUME**GM1
           TD = TDVGM1/VGM1
           TE = TEVGM1/VGM1
           TT = TTVGM1/VGM1
C
C     KEEP TRACK OF RING, PLASMA ENERGETICS.
C
           EKIN = KKIN*RSQUSQ*RLOG
           EROT = KROT*(DRSQ**2*RLOG/2.D0 + R0SQ*(DRSQ + (R1SQ + R2SQ)/
     1         4.D0))
           EMAG2=LNGTH(JMAX1)*B2*B2*(RWSQ-R2SQ)/8.D0
           EMAG=0.5D7*IRING*PHI
           DO 16 J=1,JMAX1
           EMAG1(J)=0.5D7*ILINER(J)*PSI1(J)
           EMAG=EMAG+EMAG1(J)
16         CONTINUE
```

```
      EMAG=EMAG+EMAG2
      WD = MD*VD**2/(2.D0*BOLTZ)
      WT = MT*VT**2/(2.D0*BOLTZ)
      EDDIR = NDTOT*BOLTZ*WD
      ETDIR = NTTOT*BOLTZ*WT
      EDTH = KE*NDTOT*TD
      ETTH = KE*NTTOT*TT
      EETH = KE*NETOT*TE
      EBEAM = EDDIR + EDTH
      EPLAS = ETDIR + ETTH + EETH
      ERING = EBEAM + EPLAS
      EPOT=ERING+EMAG
      ETOT = EKIN + EROT + EPOT
      IF (ETOTO .EQ. 0.D0)  ETOTO = ETOT
      YIELD = KYIELD*BURNUP
      Q = YIELD/ETOTO
      ENET = ETOT + ERAD + EOHMIC - YIELD
      PD = BOLTZ*ND*TD
      PE = BOLTZ*NE*TE
      PT = BOLTZ*NT*TT
      PTOT = PD + PE + PT
      BETA = KBETA*DSQRT(TE)
      BP = .2D0*IRING/RMINOR
      KAPPA = KPROP*VD/(RMINOR*BP)
      PPOL = KP*BP**2
      BETAPL = NKT(RMINOR, RMAJOR)/PPOL
      TL = EOHMIC/HTCAP + TLO
C
C     FIND ACCELERATION AT INNER, OUTER FACE OF LINER.
C
      O1 = OMEGA*R10SQ/R1SQ
      O2 = OMEGA*R20SQ/R2SQ
      G1 = (DY(3) - U1*U1)/R1 - O1*O1*R1
      G2 = (DY(3) - U2*U2)/R2 - O2*O2*R2
C
C     ZERO DERIVATIVES OF ION RING AND PLASMA QUANTITIES CONSERVED IN
C     THE ABSENCE OF DISSIPATION.
C
      JTOTAL=25
      DO 15 J=4, JTOTAL
   15 DY(J) = 0.D0
      JTOTAL=22
      VDT = VD - VT
      IF (NOCOLL)  GO TO 20
C
C     PUT IN COLLISIONAL EFFECTS, IF ANY.  START BY STORING THE RELATIVE
C     VELOCITIES AND THEIR SQUARES.
C
      VDE = VD - VE
      VET = VE - VT
      VTD = -VDT
      VED = -VDE
      VTE = -VET
      VDTSQ = VDT*VDT
      VDESQ = VDE*VDE
      VETSQ = VET*VET
C
      NDMD = ND*MD
      NEME = NE*ME
      NTMT = NT*MT
C
C     FIND COULOMB LOGARITHMS.
C
      LOGLDT = LOGDTO - .5D0*DLOG(NE/(TE*VDTSQ**2))
      ELOG = .5D0*DLOG(NE/TE**3)
      LOGLDE = LOGDEO - ELOG
      LOGLTE = LOGTEO - ELOG
C
```

```
C     CALCULATE THE EFFECTS OF COLLISIONS BETWEEN BEAM AND TARGET IONS.
C     THE LOW-TEMPERATURE LIMIT OF TRUBNIKOV'S FORMULAS IS USED.
C
         NUSDT = CNUSDT*LOGLDT*NT/DABS(VDT**3)
         NUSTD = NUSDT*NDMD/NTMT
         DTDST = CNUTDT*LOGLDT*NT/DABS(VDT)
         NUTDT = DTDST/TD
         DTTSD = DTDST*NDMD/NTMT
         NUTTD = DTTSD/TT
         DWDST = DTDST - KTD*NUSDT*VD*VDT
         DWTSD = DTTSD - KTT*NUSTD*VT*VTD
         IF (U1 .NE. 0.D0)  RNUBYU = RMAJOR*NUSDT/U1
C
C     USE SLOW-BEAM LIMIT TO COMPUTE INTERACTION BETWEEN IONS AND HOT
C     ELECTRONS.
C
         EFACTR = NE/TE**1.5D0
         NUSDE = CNUSDE*LOGLDE*EFACTR
         NUTDE = CNUTDE*LOGLDE*EFACTR
         DTDSE = NUTDE*(TE - TD)
         NUSTE = CNUSTE*LOGLTE*EFACTR
         NUTTE = CNUTTE*LOGLTE*EFACTR
         DTTSE = NUTTE*(TE - TT)
C
C     CONSERVATION OF MOMENTUM GIVES THE REMAINING NUS'S.
C
         NUSED = NUSDE*NDMD/NEME
         NUSET = NUSTE*NTMT/NEME
C
C     USING KNOWN NUS'S AND DT'S, THE CORRESPONDING DW'S ARE FOUND.
C
         DWDSE = DTDSE - KTD*NUSDE*VD*VDE
         DWTSE = DTTSE - KTT*NUSTE*VT*VTE
C
C     CONSERVATION OF ENERGY GIVES THE REMAINING THREE DW'S.
C
         DWESD = -DWDSE*ND/NE
         DWEST = -DWTSE*NT/NE
C
C     FROM THE DERIVED NUS'S AND DW'S, FIND THE REMAINING DT'S.
C
         DTESD = DWESD + KTE*NUSED*VE*VED
         NUTED = DTESD/TE
         DTEST = DWEST + KTE*NUSET*VE*VET
         NUTET = DTEST/TE
C
C     ADD COLLISIONAL CORRECTIONS TO THE DERIVATIVES CALCULATED ABOVE.
C
         DY(JTOTAL) =KPHI*RMAJOR*(NUSED*VED+NUSET*VET)
         DY(7) = -RMAJOR*(NUSDE*VDE + NUSDT*VDT) - KVD*DY(JTOTAL)
         DY(8) = -RMAJOR*(NUSTD*VTD + NUSTE*VTE) - KVT*DY(JTOTAL)
         DY(9) = VGM1*(DTDSE + DTDST)
         DY(10) = VGM1*(DTESD + DTEST)
         DY(11) = VGM1*(DTTSD + DTTSE)
  20     IF (NOLOSS)  GO TO 30
C
C     PUT IN OTHER SOURCES OR SINKS, IF ANY.  START WITH OHMIC LOSSES.
C
         OHMIC=0.D0
         RCONST=KRES*ETA/RLOG
         DO 25 J=1,JMAX1
         RLINER(J)=RCONST/LNGTH(J)
         OHMHT=RLINER(J)*ILINER(J)
         DY(19+J)=-OHMHT
25       OHMIC=OHMIC+OHMHT
C/OVRT   DY(5) = DPSI2
```

```
      DY(14) = 0.D0
      DO 26 J=1,JMAX1
26    DY(14) = DY(14) + 1.D7*OHMHT*ILINER(J)
C
C     THEN TREAT LOSSES BY DIFFUSION, CHARGE EXCHANGE, ETC.
C
      DY(10) = DY(10) + TELOSS
      DY(12) = NDLOSS
      DY(13) = NTLOSS
C
C     FINALLY, COMPUTE BREMSSTRAHLUNG AND CYCLOTRON RADIATION LOSSES.
C
      PCYCL = KCYCL*(BETA*BP)**2/(1.D0 - BETA**2)
      PBREM = KBREM*DSQRT(TE)*(ND*ZD*ZD + NT*ZT*ZT)
      PRAD = PBREM + PCYCL
      DTERAD = KT*PRAD
      NURAD = DTERAD/TE
      DY(15) = NETOT*PRAD
      DY(10) = DY(10) - DTERAD*VGM1
 30   IF (NOBURN)  GO TO 40
C
C     COMPUTE THERMONUCLEAR BURN.
C
      RATE = SIGMA(1.D3*WD)*DABS(VDT)*ND*NTTOT
      DY(12) = DY(12) - RATE
      DY(13) = DY(13) - RATE
      DY(16) = KNEUTR*RATE
      DY(17) = KBURN*VGM1*RATE
      DY(18) = RATE
 40   IF (1 .GT. 0)  RETURN
C
      DO 35 J=4,JTOTAL
35    DY(J)=-DY(J)
      RETURN
C
C     PASS IN DATA NEEDED TO INITIALIZE RUN, SAVING INITIAL VALUES AS Y0.
C     PRECOMPUTE CONSTANTS FOR USE IN SUBSEQUENT CALLS TO DERIV.  CGS
C     UNITS ARE USED THROUGHOUT, EXCEPT THAT TEMPERATURES AND PARTICLE
C     ENERGIES ARE IN KEV AND CIRCUIT QUANTITIES (CURRENTS, FLUXES AND
C     INDUCTANCES) ARE IN MKS.
C
      ENTRY INPUT(TIME,
     1     R10, R20, RW, B20, RHO0, OMEGA0, LNGTH0, ETA0, SPHT,
     2     TL00, PCA,
     3     RMAJ0, WD0, TD0, TE0, TT0, BP0, VRATIO, NRATIO, IRATIO,
     4     COLL, LOSS, BURN, FWD,
     5     DTPLOT, DTFILM, DTDUMP, TLAST, TPLOT, TFILM, TDUMP)
      LOGICAL*4 COLL, LOSS, BURN,FWD
      FORWRD=FWD
      RHO = RHO0
      OMEGA = OMEGA0
      ETA = ETA0
      TL0 = TL00
      RMAJOR = RMAJ0
      P=1.D1
      JMAX=3
      JTOTAL=22
      JMAX1=JMAX-1
      DLNGTH=LNGTH0/DFLOAT(JMAX1)
      DO 55 J=1,JMAX1
      LNGTH(J)=DFLOAT(J)*DLNGTH
55    LHLSQ(J)=(0.5D0*LNGTH(J))**2
      NOCOLL = .NOT. COLL
      NOLOSS = .NOT. LOSS
      NOBURN = .NOT. BURN
      DATA E, BOLTZ, C / 4.8032 D-10, 1.6022 D-9, 2.9979 D+10 /
```

```
      DATA MD, ME, MT / 3.3453 D-24, 9.1095 D-28, 5.0179 D-24 /
      DATA ZD, ZT / 1.D0, 2.D0 /
      DATA GAMMA / 1,666666667 D0 /
      DATA DPSI2 / 0.D0 /
      DATA TELOSS, NDLOSS, NTLOSS / 0.D0, 0.D0, 0.D0 /
      DATA O, RTEST / 1.D-6, 1.D-6 /
      DATA A1, A2, A3, A4, A5 / 2.823 D3, 2.59 D7, 3.98 D-6,
     1     1.297 D0, 6.47 D5 /
      DATA Z / 0.D0 /
      PI = 4.D0*DATAN(1.D0)
      SQRTPI = DSQRT(PI)
      RT2PI = DSQRT(2.D0*PI)
      TWOPI = 2.D0*PI
      FOURPI = 4.D0*PI
      EIGTPI=8.D0*PI
C
C     COMPILE TABLE OF MISCELLANEOUS CONSTANTS.
C
      GM1 = GAMMA - 1.D0
      KBETA = DSQRT(BOLTZ/(GM1*ME*C*C))
      KBREM = 5.35D-24
      KBURN = 18.34D3*BOLTZ*GM1
      KCNVRT = C/1.D1
      KCF = 1.D-7*MD
      KCYCL = 3.98D-16
      KD=0.25D0
      KE = BOLTZ/GM1
      KF=1.D7/(TWOPI*LNGTHO)
      KFLUX = 1.D8/PI
      KID = E*ZD/(TWOPI*KCNVRT)
      KIE=-E/(TWOPI*KCNVRT)
      KIT = E*ZT/(TWOPI*KCNVRT)
      KKIN=PI*RHO*LNGTHO/2.D0
      KMU = FOURPI*1.D-9
      MU = FOURPI*1.D-9
      KNEUTR = 0.D0
      KNKT = 1.D-7*BOLTZ
      KP = 1.D0/(8.D0*PI)
      KPHI = 1. D-8*TWOPI*ME*C/E
      KPROP = MD*C/(ZD*E)
      KR = DSQRT(3.D0*MD*BOLTZ)*C/E
      KRES=1.D2*FOURPI
      KROT=PI*RHO*OMEGA**2*LNGTHO
      KT = GM1/BOLTZ
      KTD = MD*GM1/BOLTZ
      KTE = ME*GM1/BOLTZ
      KTT = MT*GM1/BOLTZ
      KVD = 1.D8*ZD*E/(TWOPI*MD*C)
      KVT = 1.D8*ZT*E/(TWOPI*MT*C)
      KVOL = TWOPI*PI
      KYIELD = KBURN/GM1
C
C     CALCULATE CONSTANTS TO BE USED IN COLLISION RATES.
C
      LOGDEO = -DLOG(DSQRT(FOURPI/BOLTZ**3)*ZD*E**3)
      LOGDTO = -DLOG(DSQRT(FOURPI/BOLTZ)*ZD*ZT*E**3*(MD+MT)/(MD*MT))
      LOGTEO = -DLOG(DSQRT(FOURPI/BOLTZ**3)*ZT*E**3)
C
      CNUSDE = (4.D0*RT2PI/3.D0)*ZD**2*E**4*(1.D0 + MD/ME)*ME**1.5/
     1     (MD**2*BOLTZ**1.5)
      CNUSDT = 4.D0*PI*(ZD*ZT*E**2)**2*(1.D0 + MD/MT)/MD**2
      CNUSTE = (4.D0*RT2PI/3.D0)*ZT**2*E**4*(1.D0 + MT/ME)*ME**1.5/
     1     (MT**2*BOLTZ**1.5)
C
      CNUTDE = (8.D0*RT2PI/3.D0)*(ZD*E*E)**2*DSQRT(ME)/(MD*BOLTZ**
     1     1.5D0)
```

```
          CNUTDT = FOURPI*(ZD*ZT*E*E)**2*GM1/(BOLTZ*MD)
          CNUTTE = (8.D0*RT2PI/3.D0)*(ZT*E*E)**2*DSQRT(ME)/(MT*BOLTZ**
     1         1.5D0)
C
C     DETERMINE LINER CONSTANTS.
C
          R10SQ = R10*R10
          R20SQ = R20*R20
          R0SQ = R20SQ - R10SQ
          RWSQ = RW*RW
          HTCAP=SPHT*PI*R0SQ*LNGTH0*RHO
          PSI20 = B20*(RWSQ - R20SQ)/KFLUX
C
C     DETERMINE INITIAL RING AND PLASMA PARAMETERS.
C
          VD0 = -DSQRT(2.D0*BOLTZ*WD0/MD)
          C1 = PI*RMAJOR/1.D2
          C2 = 5.D0*BP0
          C3 = BOLTZ*(TD0 + NRATIO*TT0 + (ZD + NRATIO*ZT)*TE0)
          C4 = (KID + KIT*NRATIO*VRATIO)*(1.D0 - IRATIO)*VD0/RMAJ0
          IRING = C3/(C1*C4)
          RMIN0 = -IRING/C2
          RMINOR = RMIN0
          VOLUME = KVOL*RMAJOR*RMINOR**2
          VGM1 = VOLUME**GM1
          NDTOT0 = IRING/C4
          ND0 = NDTOT0/VOLUME
          NTTOT0 = NRATIO*NDTOT0
          NT0 = NTTOT0/VOLUME
          VT0 = VRATIO*VD0
          ID0 = KID*NDTOT0*VD0/RMAJ0
          IT0 = KIT*NTTOT0*VT0/RMAJ0
          IE0 = IRING - ID0 - IT0
          RJ0 = R10
C
C     THE INITIAL VALUES OF THE LINER CURRENTS AND NKT ARE NOW FOUND
C     THROUGH A SERIES OF ALGEBRAIC EQUATIONS DESIGNED TO INSURE
C     THAT ZERO FLUX THREADS PERPENDICULAR TO THE LINER AT
C     SPECIFIED POINTS.
C
          JM1=JMAX-2
          DO 41 J=1,JMAX
          DO 41 JJ=1,JMAX
41        ZA(J,JJ)=0.D0
          DO 44 J=1,JM1
          ZA(J,J)=KMU*DLOG((LNGTH(J+1)+LNGTH(J))/(LNGTH(J+1)
     1       -LNGTH(J)))/(TWOPI*LNGTH(J))
          IF(J.EQ.1) GO TO 44
          JP=J-1
          DO 43 JJ=1,JP
          D = (LNGTH(J+1) - LNGTH(JJ)) / 2.D0
          F = LNGTH(JJ)
43        ZA(J,JJ) = (KMU/(TWOPI*LNGTH(JJ))) * DLOG((D+F) / D)
44        CONTINUE
          DO 51 J=1,JMAX1
          ZA(JMAX,J)=DBMLRS(RMIN0,RMAJ0,RJ0,LNGTH(J),LHLSQ(J))
51        ZA(JMAX1,J)=DAMLRS(RMIN0,RMAJ0,RJ0,LNGTH(J),LHLSQ(J))
          ZA(JMAX1,JMAX)=2.D0/(RMIN0*IRING)
          ZA(JMAX,JMAX)=1.D0/(RMAJ0*IRING)
          DO 52 J=1,JM1
          DENOM=(LNGTH(J+1)/2.D0)**2+(RJ0-RMAJ0)**2
          ZR(J)=-(KMU*IRING/FOURPI)*LNGTH(J+1)/DENOM
52        CONTINUE
          ZR(JMAX1)=-0.5D0*IRING*DALR(RMIN0,RMAJ0)
          ZR(JMAX)=-0.5D0*IRING*DBLR(RMIN0,RMAJ0)-KCF*NDTOT0*VD0**2/
     1       (RMAJ0*IRING)
```

```
      JMAX2=JMAX**2
      CALL DGELG(ZR,ZA,JMAX,JMAX2,1,1.D-6,JIER)
      PRINT 42, JIER
42    FORMAT(' ',1X,'IERROR=', I2)
      PRINT 46, (ZR(J),J=1,JMAX)
46    FORMAT(1P5E12.4)
47    CONTINUE
      DO 53 J=1,JMAX1
53    ILINER(J)=ZR(J)
      NKTO=ZR(JMAX)
C
      PHIO=LR(RMINO,RMAJO)*IRING
      DO 54 J=1,JMAX1
      D=LNGTH(J)
      F=LHLSQ(J)
      PHIO=PHIO+MLRS(RMINO,RMAJO,RJO,D,F)*ILINER(J)
      PSI1(J)=LL(RJO,D)*ILINER(J)+MLRS(RMINO,RMAJO,RJO,D,F)*IRING
      DO 62 JJ=1,JMAX1
      IF(J.EQ.JJ) GO TO 62
      PSI1(J) = PSI1(J) + MLL(RJO,LNGTH(J),LNGTH(JJ))*ILINER(JJ)
62    CONTINUE
      Y(J+19)=PSI1(J)
54    CONTINUE
      NUO = CNUSDT*2.D1*NTO/(DABS(VDO - VTO))**3
      U1O = RMAJOR*NUO/PCA
      IF (FORWRD)  U1O = -U1O
C
C     INITIALIZE THE DEPENDENT VARIABLE (Y) ARRAY.
C
      Y(1) = TIME
      Y(2) = R1OSQ
      Y(3) = R1O*U1O
      Y(4) = 0.DO
      Y(5) = PSI2O
      Y(6) = 0.DO
      Y(7) = RMAJO*VDO
      Y(8) = RMAJO*VTO
      Y(9) = TDO*VGM1
      Y(10) = TEO*VGM1
      Y(11) = TTO*VGM1
      Y(12) = NDTOTO
      Y(13) = NTTOTO
      Y(14) = 0.DO
      Y(15) = 0.DO
      Y(16) = 0.DO
      Y(17) = 0.DO
      Y(18) = 0.DO
      Y(19) = 0.DO
      DO 60 J=1,JMAX1
60    Y(19+J)=PSI1(J)
      Y(JTOTAL)=PHIO
C
C     COPY INITIAL VALUES INTO SAVE ARRAY.
C
      DO 50 J=1,JTOTAL
  50  YO(J) = Y(J)
C
C     INITIALIZE EXTRAPOLATION PROCEDURE FOR FIRST GUESSES USED IN
C     NEWTON-RAPHSON ROUTINE ON EVEN STEPS OF R-K-G.
C
      DO 61  J = 1, 4
      RMAJ1(J) = RMAJO
      RMAJ2(J) = RMAJO
      RMIN1(J) = RMINO
61    RMIN2(J) = RMINO
      DTNEW = DT
```

```
      RETURN
C
      ENTRY DSPLAY(SNAME)
      DATA SY, SYO, SDY, SLI, SRING, SPLAS, SNU, SCNU, SDATA /
     1     'Y   ', 'YO  ', 'DY  ', 'LI  ', 'RING', 'PLAS', 'NU  ',
     2     'CNU ', 'DATA' /
      DATA SBLANK / '    ' /
      DO 100  J = 1, 33
 100  SLINE(J) = SBLANK
C
      IF (SNAME .EQ. SY   )  GO TO 110
      IF (SNAME .EQ. SYO  )  GO TO 120
      IF (SNAME .EQ. SDY  )  GO TO 130
      IF (SNAME .EQ. SLI  )  GO TO 140
      IF (SNAME .EQ. SRING)  GO TO 150
      IF (SNAME .EQ. SPLAS)  GO TO 160
      IF (SNAME .EQ. SNU  )  GO TO 170
      IF (SNAME .EQ. SCNU )  GO TO 180
      IF (SNAME .EQ. SDATA)  GO TO 190
      PRINT 101, SNAME
 101  FORMAT('0', 'DSPLAY CALLED WITH INVALID ARGUMENT ', A4 //)
      RETURN
C
C     DEPENDENT VARIABLES.
C
 110  PRINT 111
 111  FORMAT('-  Y(1)     Y(2)     Y(3)     Y(5)     Y(6)     Y(7)    ',
     1     'Y(9)     Y(10)    Y(11)    Y(12)    Y(13)    Y(14)    Y(15)',
     2     '    Y(16)    Y(17)    Y(18)')
      CALL SCNVR5(SLINE, Y(1), Y(2), Y(3), Y(5), Y(6), Y(7), Y(9),
     1     Y(10), Y(11), Y(12), Y(13), Y(14), Y(15), Y(16), Y(17),
     2     Y(18))
      PRINT 199, SLINE
      RETURN
C
C     INITIAL VALUES OF DEPENDENT VARIABLES.
C
 120  PRINT 121
 121  FORMAT('-  YO(1)    YO(2)    YO(3)    YO(5)    YO(6)    YO(7)   ',
     1     'YO(9)    YO(10)   YO(11)   YO(12)   YO(13)   YO(14)   YO(15)',
     2     '   YO(16)   YO(17)   YO(18)')
      CALL SCNVR5(SLINE, YO(1), YO(2), YO(3), YO(5), YO(6), YO(7),
     1     YO(9), YO(10), YO(11), YO(12), YO(13), YO(14), YO(15),
     2     YO(16), YO(17), YO(18))
      PRINT 199, SLINE
      RETURN
C
C     DERIVATIVES OF DEPENDENT VARIABLES.
C
 130  PRINT 131
 131  FORMAT('-  DY(1)    DY(2)    DY(3)    DY(5)    DY(6)    DY(7)   ',
     1     'DY(9)    DY(10)   DY(11)   DY(12)   DY(13)   DY(14)   DY(15)',
     2     '   DY(16)   DY(17)   DY(18)')
      CALL SCNVR5(SLINE, DY(1), DY(2), DY(3), DY(5), DY(6), DY(7),
     1     DY(9), DY(10), DY(11), DY(12), DY(13), DY(14), DY(15),
     2     DY(16), DY(17), DY(18))
      PRINT 199, SLINE
      RETURN
C
C     LINER PARAMETERS AND ENERGETICS.
C
 140  PRINT 141
 141  FORMAT('-   R1       R2       U1       U2       G1       G2     ',
     1     '  B1       B2      ETA      PSI2    RSKIN  PSI1(1) PSI1(2)',
     2     ' PSI1(3) PSI1(4) PSI1(5)')
```

```
      CALL SCNVRS(SLINE, [illegible]
     1     PSI2, RJ, PSI1(1), PSI1(2), PSI1(3), PSI1(4), PSI1(5))
      PRINT 199, SLINE
      PRINT 142
 142  FORMAT('- ETOT    EKIN    EROT    EPOT    EMAG    EMAG2   ',
     1     'ERING  EBEAM   EDDIR   EDTH    EPLAS   EETH    ETTH   ',
     1     ' ETDIR   ENET       Q')
      CALL SCNVRS(SLINE, ETOT, EKIN, EROT, EPOT, EMAG, EMAG2,
     1     ERING, EBEAM, EDDIR, EDTH, EPLAS, EETH, ETTH, ETDIR,
     1     ENET, Q)
      PRINT 199, SLINE
      PRINT 143
 143  FORMAT('- EMAG(1) EMAG(2) EMAG(3) EMAG(4) EMAG(5)   IL1     ',
     1     'IL2     IL3     IL4     IL5     RL1     RL2     RL3    ',
     2     ' RL4     RL5     TL')
      CALL SCNVRS(SLINE, EMAG1(1), EMAG1(2), EMAG1(3), EMAG1(4),
     1     EMAG1(5), ILINER(1), ILINER(2), Z,Z,Z,
     2      RLINER(1), RLINER(2), RLINER(3), RLINER(4),
     3     RLINER(5), TL)
      PRINT 199, SLINE
      RETURN
C
C     PARAMETERS OF ION AND IMAGE RINGS.
C
 150  PRINT 151
 151  FORMAT('- RMAJOR  RMINOR  IRING   ID      LRING   MLR1    ',
     1     ' PHI     PD      VD      WD      NDTOT   ND      TD     ',
     1     'VOLUME   ITER    RATE')
      CALL SCNVRS(SLINE, RMAJOR, RMINOR, IRING, ID, LRING, MLR1S,
     1     PHI, PD, VD, WD, NDTOT, ND, TD, VOLUME, ITER, RATE)
      PRINT 199, SLINE
      RETURN
C
C     TARGET PLASMA AND RADIATION PARAMETERS.
C
 160  PRINT 161
 161  FORMAT('- NETOT   NTTOT  VOLUME   NE      NT      WT      ',
     1     'VE      VT      PTOT    TE      TT      PE      PT    ',
     2     '   IE      IT     YIELD')
      CALL SCNVRS(SLINE, NETOT, NTTOT, VOLUME, NE, NT, WT, VE, VT,
     1     PTOT, TE, TT, PE, PT, IE, IT, YIELD)
      PRINT 199, SLINE
      PRINT 162
 162  FORMAT('- BETA    BP      PCYCL   PBREM   PRAD    NURAD  ',
     1     ' ERAD   DTERAD   PPOL    RNUBYU  NCOUNT  BETAPL  EOHMIC ',
     2     '  KAPPA  PVGPRD  BURNUP')
      CALL SCNVRS(SLINE, BETA, BP, PCYCL, PBREM, PRAD, NURAD, ERAD,
     1     DTERAD, PPOL, RNUBYU, NCOUNT, BETAPL, EOHMIC, KAPPA,
     2     PVGPRD, BURNUP)
      PRINT 199, SLINE
      RETURN
C
C     TRANSPORT RATES.
C
 170  PRINT 171
 171  FORMAT('- NUSDE   NUSDT   NUSTE   NUSED   NUSTD   NUSET   ',
     1     ' VDT     VDE     VET     ELOG    DWD/E   DWD/T   DWT/E ',
     2     '  DWE/D   DWT/D   DWE/T')
      CALL SCNVRS(SLINE, NUSDE, NUSDT, NUSTE, NUSED, NUSTD, NUSET,
     1     VDT, VDE, VET, ELOG, DWDSE, DWDST, DWTSE, DWESD, DWTSD,
     2     DWEST)
      PRINT 199, SLINE
      PRINT 172
 172  FORMAT('- NUTDE   NUTDT   NUTET   NUTED   NUTTD   NUTTE   ',
     1     'DTD/E   DTD/T   DTE/T   DTE/D   DTT/D   DTT/E  LOGLDE',
     2     '  LOGLDT  LOGLTE')
      CALL SCNVRS(SLINE, NUTDE, NUTDT, NUTET, NUTED, NUTTD, NUTTE,
```

```
     1          DTDSE, DTDST, DTEST, DTESD, DTTSD, DTTSE, LOGLDE, LOGLDT,
     2          LOGLTE, Z)
          PRINT 199, SLINE
C
C     INDUCTANCES.
C
          PRINT 173
173       FORMAT('=   LL1      LL2      LL3      LL4     LL5   THIS SPACE',
     1          ' FOR RENT ')
          CALL SCNVRS(SLINE, LL1, LL2, LL3, LL4, LL5, Z, Z , Z, Z, Z,
     1          Z, Z, Z, Z, Z, Z)
          PRINT 199, SLINE
          PRINT 175
175       FORMAT('=  LL5     ML12     ML13     ML14     ML15     ML23     ',
     1          'ML24     ML25     ML34     ML35     ML45     MLRS1    MLRS2 ',
     2          '  MLRS3   MLRS4    MLRS5 ')
          CALL SCNVRS(SLINE, LL5, ML12, ML13, ML14, ML15, ML23, ML24,
     1          ML25, ML34, ML35, ML45, MLRS1, MLRS2, MLRS3, MLRS4, MLRS5)
          PRINT 199, SLINE
          RETURN
C
C     CONSTANTS USED IN EVALUATING TRANSPORT AND RADIATION RATES.
C
 180      PRINT 181
 181      FORMAT('= LOGDEO  LOGDTO  LOGTEO    KTD      KTE      KTT    ',
     1          'CNUSDE  CNUSDT  CNUSTE  CNUTDE  CNUTDT  CNUTTE   KBETA ',
     2          '  KBREM   KCYCL    KT')
          CALL SCNVRS(SLINE, LOGDEO, LOGDTO, LOGTEO, KTD, KTE, KTT,
     1          CNUSDE, CNUSDT, CNUSTE, CNUTDE,CNUTDT, CNUTTE, KBETA,
     2          KBREM, KCYCL, KT)
          PRINT 199, SLINE
          RETURN
C
C     PHYSICAL CONSTANTS AND STORED COMBINATIONS THEREOF.
C
 190      PRINT 191
 191      FORMAT('=     E      BOLTZ      C       MD       ME       MT    ',
     1          '   ZD       ZT       RW    LENGTH  TELOSS  NDLOSS  NTLOSS',
     2          '   DPSI2   OMEGA    GAMMA')
          CALL SCNVRS(SLINE, E, BOLTZ, C, MD, ME, MT, ZD, ZT, RW,
     1          LNGTHO, TELOSS, NDLOSS, NTLOSS, DPSI2, OMEGA, GAMMA)
          PRINT 199, SLINE
          PRINT 192
 192      FORMAT('=  KBURN   KCNVRT    KCF      KD       KE       KF     ',
     1          'KFLUX     KID      KIE      KIT     KKIN     KMU    KNEUTR ',
     2          ' KNKT     KP      KPROP')
          CALL SCNVRS(SLINE, KBURN, KCNVRT, KCF, KD, KE, KF, KFLUX,
     1          KID, KIE, KIT, KKIN, KMU, KNEUTR, KNKT, KP, KPROP)
          PRINT 199, SLINE
          PRINT 193
 193      FORMAT('=  KPHI     KR      KRES     KROT     KVD      KVT    ',
     1          ' KVOL     KVD      KVT    KYIELD     DR     RTEST    SPHT  ',
     2          '  HTCAP')
          CALL SCNVRS(SLINE, KPHI, KR, KRES, KROT, KVD, KVT, KVOL, KVD,
     1          KVT, KYIELD, D, RTEST, SPHT, HTCAP, Z, Z)
          PRINT 199, SLINE
          PRINT 194
 194      FORMAT('=   DT    DTPLOT  DTFILM  DTDUMP    TLAST      T     ',
     1          'TPLOT    TFILM   TDUMP   IRATIO  VRATIO  NRATIO')
          CALL SCNVRS(SLINE, DT, DTPLOT, DTFILM, DTDUMP, TLAST, TIME,
     1          TPLOT, TFILM, TDUMP, IRATIO, VRATIO, NRATIO, Z, Z, Z, Z)
          PRINT 199, SLINE
          RETURN
C
 199      FORMAT(' ', 33A4)
```

```
C
      ENTRY OUTPUT
         RETURN
         E N D
C
         REAL FUNCTION FA*8(A,B,C,NKT)
         IMPLICIT REAL*8(A-I,K-Z), INTEGER*4(J)
         DIMENSION YA(3,3)
         COMMON / INDEX / JMAX, JMAX1
         COMMON / PBLOCK / P
         COMMON/ RADII/ RMINOR,RMAJOR,RJ
         COMMON/ LENGTH/  LNGTH(5), LHLSQ(5)
         COMMON/ ARRAY/ Y(25), DY(25), YO(25)
         COMMON / INCO / MU, KMU
         COMMON/ FLUX/ PHI,PSI1(5)
         COMMON / CURREN / R(3)
         LR(A,B) = KMU*B*(DLOG(8.D0*B/A) - 2.D0 + KD)
         DALR(A,B) = -KMU*B/A
         DBLR(A,B) = KMU*(DLOG(8.D0*B/A) - 1.D0 + KD)
         LL(C,D)=MU*C*(DLOG(16.D0*C/D) - 0.5D0)
         DCLL(C,D)=MU*(DLOG(16.D0*C/D) + 0.5D0)
         LN(D,F)=0.5D0*(D/F)*DLOG(D)
         FAC(D,F)=(((F-D)**2)/(2.D0*D*F))*DLOG(DABS(F-D))
         MLL(C,D,F)=MU*C*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)-0.5D0)
         DCMLL(C,D,F)=MU*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)+0.5D0)
         MLR(B,C,F)=MU*DSQRT(B*C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2 +
     1        F)) - 1.D0 - ((B-C)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
         DBMLR(B,C,F)=.5D0*MU*DSQRT(C/B)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1        +F))+((C-3.D0*B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
         DCMLR(B,C,F)=.5D0*MU*DSQRT(B/C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1        +F))+((3.D0*C-B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
         MLRS(A,B,C,D,F)=MLR(B,C,F)+(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))
     1        /(1.D0+((C-B)/A)**P)
         DAMLRS(A,B,C,D,F)=(.5D0*DALR(A,B)*DSQRT(LL(C,D)/LR(A,B))+(P/A)
     1       *((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))/(1.D0+
     2        ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
         DBMLRS(A,B,C,D,F)=DBMLR(B,C,F)+(.5D0*DBLR(A,B)*DSQRT(LL(C,D)/
     1        LR(A,B)) + (P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-
     2      MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
         DCMLRS(A,B,C,D,F)=DCMLR(B,C,F)+(.5D0*DCLL(C,D)*DSQRT(LR(A,B)/
     1        LL(C,D))-(P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B  )*LL(C,D))-
     2      MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
C     A SERIES OF ALGEBRAIC EQUATIONS IS SOLVED TO FIND THE CURRENTS
C     IN THE LINER AND RING.
         KD = .25D0
         MINR=A
         MAJR=B
         LIR=C
         LIR2=LIR
         JMAXM=JMAX-1
         DO 1 J=1,JMAXM
    1    R(J)=Y(19+J)
         R(JMAX)=Y(19+JMAX)
         YA(1,1)= LL(LIR,LNGTH(1))
         DO 2 J1=2,JMAXM
         A1=LNGTH(J1)
        YA(J1,J1)=LL(LIR,A1)
         J1M1=J1-1
         DO 2 J2=1,J1M1
         F=LNGTH(J2)
        YA(J1,J2)=MLL(LIR,A1,F)
         YA(J2,J1)=YA(J1,J2)
    2    CONTINUE
         DO 4 J=1,JMAXM
         A1=LNGTH(J)
         D=LHLSQ(J)
```

```
      YA(JMAX,J)=MLRS(MINR,MAJR,LIR,A1,D)
      YA(J,JMAX)=YA(JMAX,J)
      YA(JMAX,JMAX)=LR(MINR,MAJR)
      JMAX2=JMAX**2
      CALL DGELG (R, YA, JMAX, JMAX2, 1, 1.D-6, JIER)
      FA          =0.5D0*R(JMAX)**2*DALR(A,B)+2.D0*NKT/A
      DO 20 J=1,JMAX1
      D=LNGTH(J)
      F=LHLSQ(J)
20    FA          =FA          +R(JMAX)*R(J)*DAMLRS(A,B,C,D,F)
      RETURN
      END
      REAL FUNCTION FB*8(A,B,C,NKT)
      IMPLICIT REAL*8(A-I,K-Z), INTEGER*4(J)
      DIMENSION YA(3,3)
      COMMON / INCO / MU, KMU
      COMMON/ COEFF/ KCF,NDTOT, RVD
      COMMON / PBLOCK / P
      COMMON / CURREN / R(3)
      COMMON/ LENGTH/  LNGTH(5), LHLSQ(5)
      COMMON / INDEX / JMAX, JMAX1
      COMMON/ ARRAY/ Y(25), DY(25), YO(25)
      COMMON/ RADII/ RMINOR,RMAJOR,RJ
      COMMON/ FLUX/ PHI,PSI1(5)
      LR(A,B) = KMU*B*(DLOG(8.D0*B/A) - 2.D0 + KD)
      DALR(A,B) = -KMU*B/A
      DBLR(A,B) = KMU*(DLOG(8.D0*B/A) - 1.D0 + KD)
      LL(C,D)=MU*C*(DLOG(16.D0*C/D) - 0.5D0)
      DCLL(C,D)=MU*(DLOG(16.D0*C/D) + 0.5D0)
      LN(D,F)=0.5D0*(D/F)*DLOG(D)
      FAC(D,F)=(((F-D)**2)/(2.D0*D*F))*DLOG(DABS(F-D))
      MLL(C,D,F)=MU*C*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)-0.5D0)
      DCMLL(C,D,F)=MU*(DLOG(16.D0*C)-LN(D,F)-LN(F,D)+FAC(D,F)+0.5D0)
      MLR(B,C,F)=MU*DSQRT(B*C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2 +
     1     F)) - 1.D0 - ((B-C)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      DBMLR(B,C,F)=.5D0*MU*DSQRT(C/B)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1     +F))+((C-3.D0*B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      DCMLR(B,C,F)=.5D0*MU*DSQRT(B/C)*(.5D0*DLOG(6.4D1*B*C/((B-C)**2
     1     +F))+((3.D0*C-B)/DSQRT(F))*DATAN(DSQRT(F)/(B-C)))
      MLRS(A,B,C,D,F)=MLR(B,C,F)+(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))
     1     /(1.D0+((C-B)/A)**P)
      DAMLRS(A,B,C,D,F)=(.5D0*DALR(A,B)*DSQRT(LL(C,D)/LR(A,B))+(P/A)
     1    *((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-MLR(B,C,F))/(1.D0+
     2     ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
      DBMLRS(A,B,C,D,F)=DBMLR(B,C,F)+(.5D0*DBLR(A,B)*DSQRT(LL(C,D)/
     1     LR(A,B)) + (P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B)*LL(C,D))-
     2    MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
      DCMLRS(A,B,C,D,F)=DCMLR(B,C,F)+(.5D0*DCLL(C,D)*DSQRT(LR(A,B)/
     1     LL(C,D))-(P/(C-B))*((C-B)/A)**P*(DSQRT(LR(A,B  )*LL(C,D))-
     2    MLR(B,C,F))/(1.D0 + ((C-B)/A)**P))/(1.D0 + ((C-B)/A)**P)
C     A SERIES OF ALGEBRAIC EQUATIONS IS SOLVED TO FIND THE CURRENTS
C     IN THE LINER AND RING.
      KD = .25D0
      MINR=A
      MAJR=B
      LIR=C
      LIR2=LIR
      JMAXM=JMAX-1
      DO 1 J=1,JMAXM
1     R(J)=Y(19+J)
      R(JMAX)=Y(19+JMAX)
      YA(1,1)= LL(LIR,LNGTH(1))
      DO 2 J1=2,JMAXM
      A1=LNGTH(J1)
      YA(J1,J1)=LL(LIR,A1)
      J1M1=J1-1
```

```
          DO 2 J2=1,JIMI
          F=LNGTH(J2)
         YA(J1,J2)=MLL(LIR,A1,F)
          YA(J2,J1)=YA(J1,J2)
2         CONTINUE
          DO 4 J=1,JMAXM
          A1=LNGTH(J)
          D=LHLSQ(J)
         YA(JMAX,J)=MLRS(MINR,MAJR,LIR,A1,D)
4         YA(J,JMAX)=YA(JMAX,J)
         YA(JMAX,JMAX)=LR(MINR,MAJR)
          JMAX2=JMAX**2
          CALL DGELG (R, YA, JMAX, JMAX2, 1, 1.D-6, JIER)
          FB=0.5D0*R(JMAX)**2*DBLR(A,B)+NKT/B+KCF*NDTOT*RVD**2/B**3
          DO 20 J=1,JMAX1
          D=LNGTH(J)
          F=LHLSQ(J)
20        FB          =FB             +R(JMAX)*R(J)*DBMLRS(A,B,C,D,F)
          RETURN
          END
```

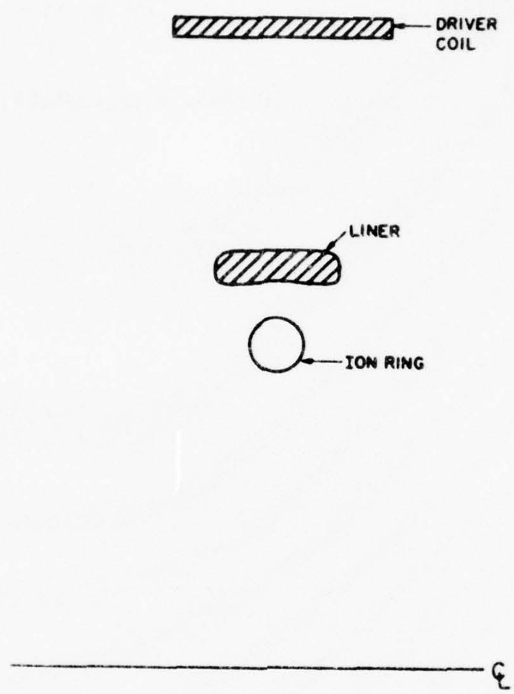

Fig. 1 – Schematic of a fixed driver coil and moving liner and ion beam plasma ring, all having finite length and roughly satisfying the large-aspect ratio approximation.

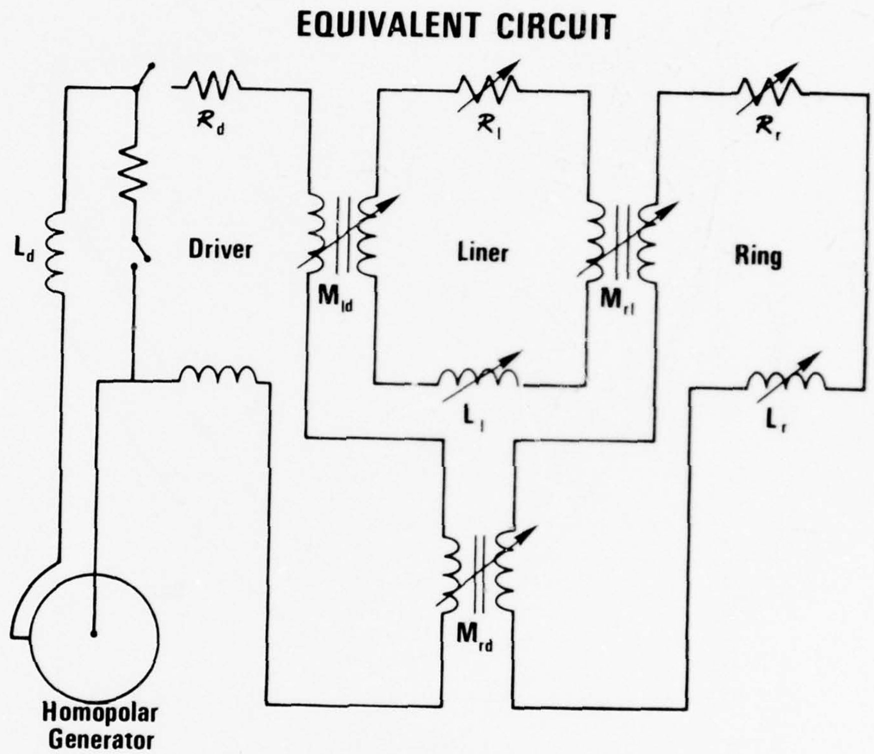

Fig. 2 – Electrical circuit equivalent to Fig. 1. A homopolar generator is used to energize the driving coil.

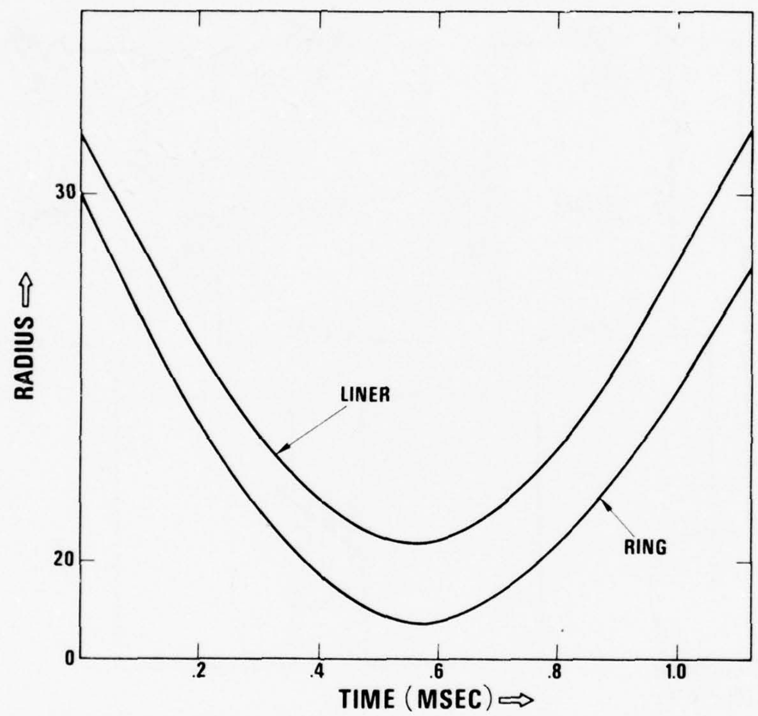

Fig. 3 – Ring and liner radii vs t for the given initial conditions.

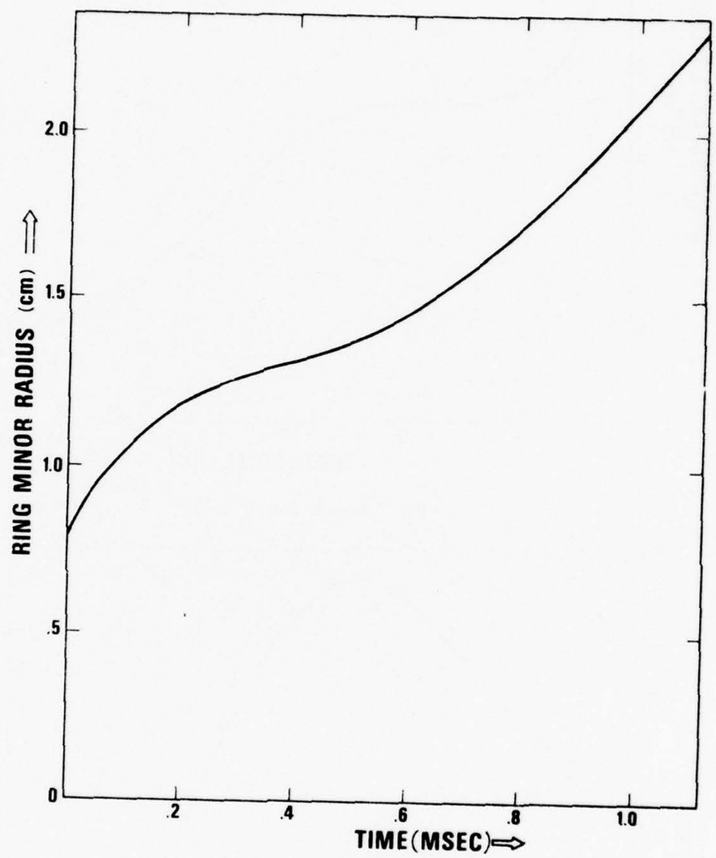

Fig. 4 – Ring minor radius vs. t.

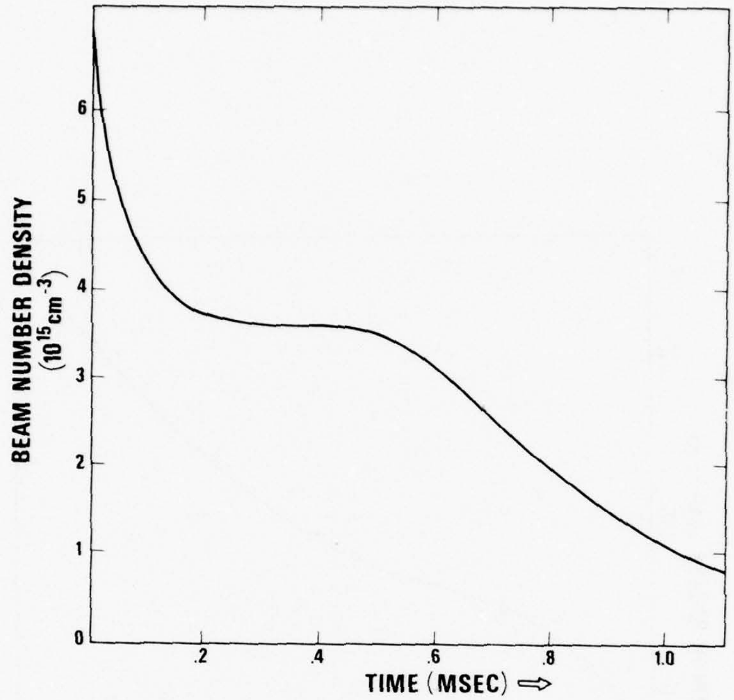

Fig. 5 – Beam number density vs. t.

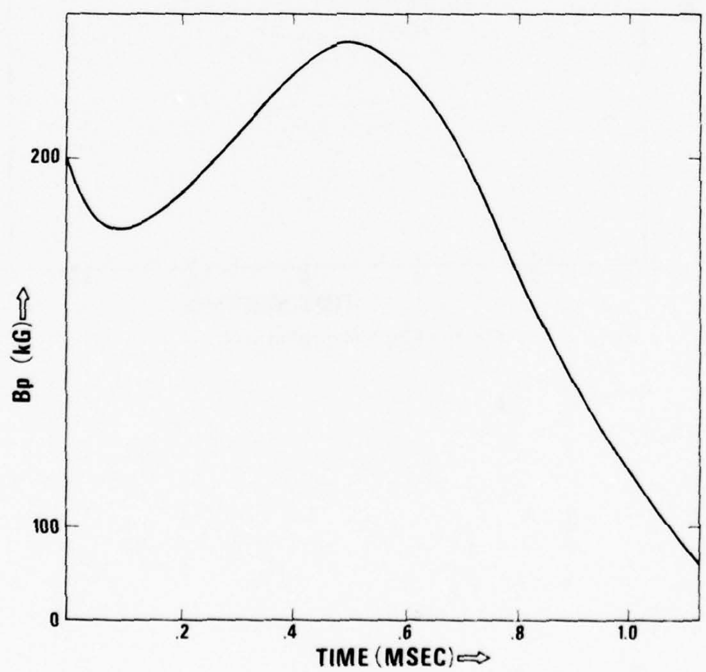

Fig. 6 – Poloidal magnetic field vs t.

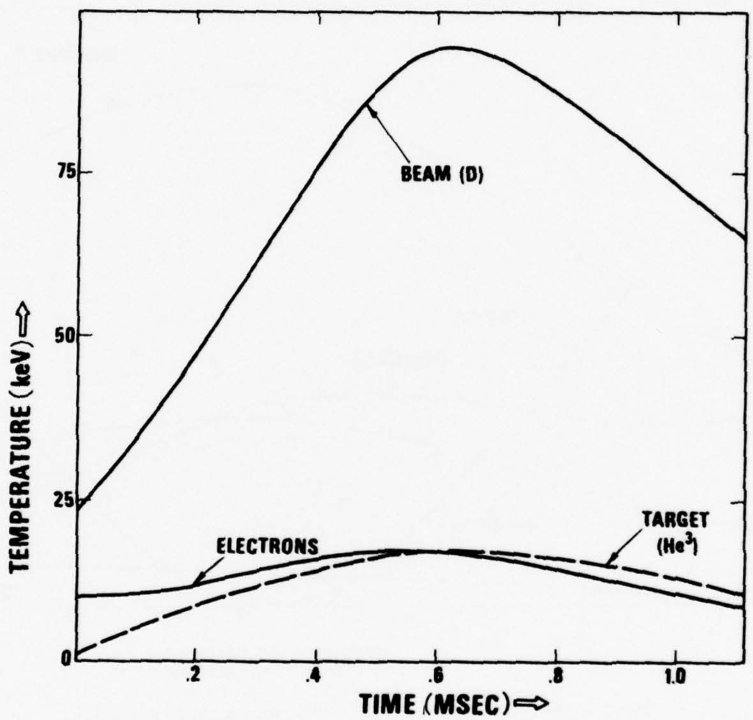

Fig. 8 – Beam, target ion and electron temperatures vs. t.

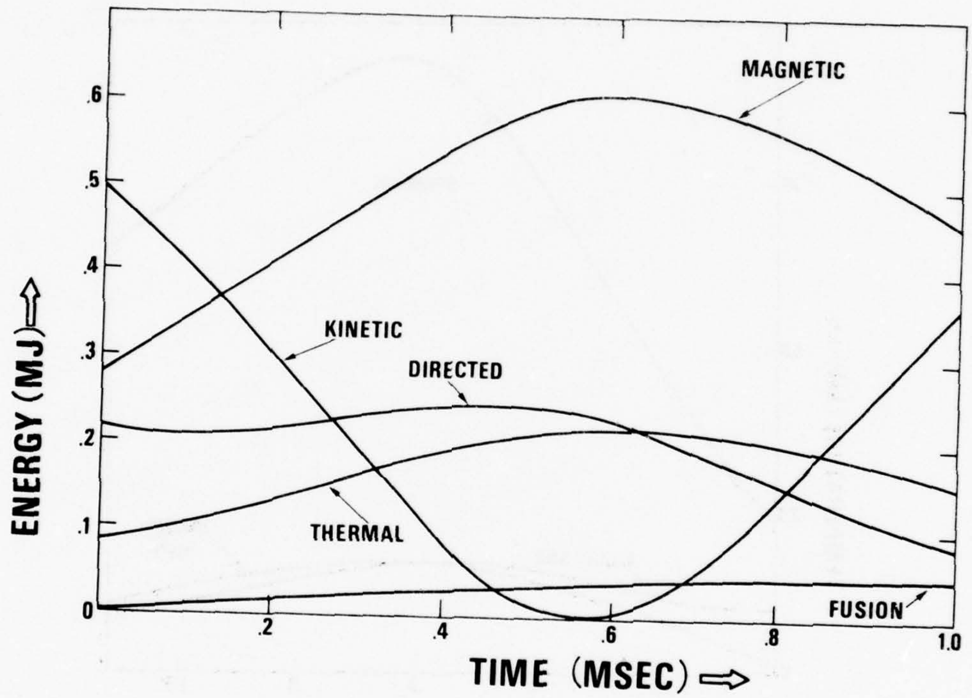

Fig. 7 – Magnetic energy [from Eq. (9)], liner kinetic energy, ion streaming energy, and total particle thermal energy vs. t.

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